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Estimating Potential Output and the Output Gap in Slovenia Using an Unobserved Components Model

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Abstract

This paper analyses the dynamics of potential output and output gap in Slovenia using the combination of production function and unobserved components model (UCM) methodology in a small semi-structural model. This combination allows potential output estimates to incorporate more economic structure than within the traditional production function approach, while it still preserves the production function as a key organising element. Despite the parsimonious structure of the framework, extended UCM is able to track the narrative on macroeconomic cycles and trends of the Slovenian economy relatively well. The applied production function methodology for estimating potential output also allows us to calculate both contributions of different unobservable drivers (trend components of TFP, capital and labour) to the overall potential output growth and the impact of main unobservable gap variables that are included in the state-space system on the output gap estimate. Regarding the long-term developments, we comment the results obtained using a set of purely technical long-term assumptions, which are mostly based on historical developments of included series (except for the labour market variables). Lastly, we also present some results of a pseudo real-time forecasting exercise, where we focus on comparing expanding window pseudo real-time forecasting ability of our model with a 4-variable (B)VAR models and on analysing pseudo real-time output gap revisions between H-P filter and extended UCM.

JEL Classification Numbers: C11, C32, E31, E32, E52

Keywords: potential output, unobserved components model, Bayesian estimation methods, pseudo real-time forecasting

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Povzetek

V članku je predstavljena analiza dinamike potencialnega proizvoda in proizvodne vrzeli v Sloveniji z uporabo kombinacije proizvodne funkcije in metodologije neopazovanih komponent (UCM) v majhnem semi-strukturnem modelu. Uporaba tovrstne kombinacije nam omogoča, da ocene potencialnega proizvoda vključujejo več ekonomske strukture kot je le-te prisotne v tradicionalnem pristopu proizvodne funkcije, hkrati pa se proizvodna funkcija še vedno ohranja kot ključni organizacijski element. Navkljub razmeroma enostavni strukturi, ocene razširjenega UCM dobro sledijo ciklični in tredni makroekonomski dinamiki slovenskega gospodarstva. Uporabljena metodologija proizvodne funkcije za oceno potencialnega proizvoda nadalje omogoča izračun tako prispevkov različnih neopazovanih dejavnikov (trendne komponente TFP, kapitala in delovne sile) k celotni rasti potencialnega proizvoda, kot tudi vpliva glavnih neopazovanih vrzelnih spremenljivk, ki so vključene v model prostora-stanja, na oceno proizvodne vrzeli. Pri analizi dolgoročne rasti potencialnega proizvoda gradivo obravnava rezultate, pridobljene z uporabo nabora tehničnih dolgoročnih predpostavk, ki večinoma temeljijo na pretekli dinamiki vključenih spremenljivk (izjema so spremenljivke na trgu dela). Zadnji del analize predstavlja rezultate napovedovanja v psevdo realnem-času. Gradivo se osredotoča na primerjavo natančnosti napovedi med razširjenim UCM in (B)VAR modeloma z uporabo metode razširjenega okna v psevdo realnem-času ter na primerjavo revizij ocen proizvodne vrzeli med H-P filtrom in razširjenim UCM v psevdo realnem-času.

1 Introduction

Potential output is generally considered to be the level of sustainable aggregate supply capabilities of an economy (determined by the structure of production, state of the technology and available inputs) at which no upward or downward pressures on inflation exists (Okun, 1962). Any deviation of the actual output from this level opens either positive or negative output gap, which, due to the existence of real and nominal rigidities, creates room for active stabilization policy. Given the specified link between output gap and inflation, the precise estimation of potential output can therefore provide information on the state of the economic cycle and the implications for the dynamics of wages and prices over the short to medium-term. In addition, potential output can also be relevant for longer-term structural analysis - for instance, in measuring the impact of structural reforms on the longer-term growth rate or estimating the natural rate of interest. The correct evaluation of its growth and precise determination of cyclical position of the economy (output gap) is therefore of crucial importance for the policy makers to be able to conduct the appropriate stabilization policy (also important element in designing European fiscal rules). As such, developing potential output estimates and forming expectations about its future developments is central to many of the current debates and thus requires addressing such challenging issue in a systematic way.

The estimation of potential output and consequent determination of the output gap is a complicated task, since neither of the two concepts is directly observable. This indicates that there exists an uncertainty related to the accuracy of any particular estimate or forecast, since their ex-post comparison with the actual data is not possible even when all observable series become available. The "usefulness" and the ability of potential output estimates to provide a reasonable macroeconomic narrative must therefore be assessed given the researcher's explicit criteria. Obtained estimates are then evaluated against these criteria. In addition to the abovementioned problem, there also exist a number of different competing estimation techniques, which range from simple univariate filtering methods to recently more popular micro founded and fully structural New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) models. Given such plurality of views on how to best interpret and estimate the unobservable measures, a potential range of different estimates can be recognized as another source of uncertainty (i.e. model uncertainty), which is always present.

Against this background, the current paper utilizes the extended unobserved components model (extended UCM) approach (similar to one applied in Beneš et al., 2010; Blagrave et al., 2015; Melolinna & Tóth, 2016; Alichi et al., 2017; Morgan et al., 2019 and Tóth, 2019), which is an extension of univariate filtering techniques to the semi-structural multivariate filtering framework for estimating potential output and output gap. There exist several reasons, why the proposed method might be relevant. First, the analysis in the state-space system could be important since the multivariate filter includes some well known economic identification restrictions, which might help in obtaining more economically plausible estimates. Second, in comparison to univariate filtering techniques, the multivariate alternative might produce more stable “end-point” estimates, simply due to incorporating more relevant economic information. Lastly, the results obtained from the multivariate method can be also adjusted in a transparent manner using information from outside of the model, which, however, is not feasible by utilizing univariate filtering techniques. This could be particularly helpful at the end of the sample, given the uncertainty of real-time estimates. Given the three advantages, the multivariate filter methods provide a very useful starting point for an analysis and at the same time put some structure on the estimation process.

The primary goal of this study is therefore to closely consider the approach of combining production function methodology with the UCM approach, as well as to compare the short to medium-term forecasting ability of our model with the performance of a 4-variable (Bayesian) vector autoregression models ((B)VAR models) in order to examine the credibility of obtained estimates. In that sense we can answer the question, whether the obtained estimates of the potential output are able to replicate the actual macroeconomic cycles and trends (especially the pre-crisis build up and prolonged recovery period afterwards), while at the same time we can analyse the relevance of the information content of the estimates for the medium-term policy analysis. To the best of our knowledge, current research is one of the first applications of the combined production function and UCM methodologies to Slovenian data[1].

The rest of the paper is organized as follows. Section 2 presents a short literature review of the most important findings of the previous research. Section 3 discusses the extended UCM modelling framework that stands behind the potential output and output gap calculation procedures. Section 4 presents the data set for Slovenia, illustrates the applied statistical techniques that are necessary to modify the data set, explains the estimation methodology that is used to obtain parameter values and demonstrates the results of the empirical exercise. Section 5 concludes.

[1]An analysis by Jemec (2012) can be considered as the most closely related research, which however treats production function and UCM techniques as two separate modelling approaches. In addition, our UCM compared to the one developed by Jemec (2012) exerts more complex structure as it, besides real GDP and price inflation, includes also unemployment rate, wage inflation, labour force participation rate, average hours worked, working age population and capital stock as an additional sources of information.

2 Literature review

First, we review the most important literature on potential output and output gap modelling. In each of the corresponding research, special attention is given to the explicit econometric methodology used and their main advantages. The review is mainly focused on the production function and conventional UCM methodologies, since both have recently gained considerable attention among central banks, mainly due to the ability of both modelling frameworks to consistently incorporate more economic structure that cannot be captured in the purely mechanic univariate filtering techniques.

Potential output and the output gap estimation was a very relevant and challenging topic of the macroeconomic research even before the existence of the more sophisticated methods. Early works of Hodrick and Prescott (1981, 1997), Beveridge and Nelson (1981), King and Rebelo (1993) and Baxter and King (1999) were mostly engaged in the application of univariate filters i.e. the methods, predominantly based on the ideas of extracting trend and cycle from output series using a purely statistical approach[2]. The advantages of this approach are its simplicity (i.e. it requires only one data series (output)) and coherency. On the other hand, due to its relative simplicity the method carries also several notable limitations. The main disadvantage is that the estimates coming out of univariate filter analyses are usually considered more of a "trend" (rather than potential) growth, since these techniques usually ignore relevant economic information and structure. In addition, the estimates coming out of these filters reflect several statistical features, which users have to be aware of. For example, in the Hodrick-Prescott filter (H-P filter) the estimates of the output gap have usually a mean reversion property, and the relative volatility of the cyclical and trend components is crucially determined by a single exogenous smoothness parameter (λ) (Blagrave et al., 2015). If the value of this parameter is set to 0, then the cyclical component is equal to 0, meaning that the trend component and actual series coincide. On the other hand, if the value of this parameter approaches ∞, the trend component more and more resembles linear deterministic trend. Nevertheless, we should point out that parameter λ is highly endogenous with respect to the parameters of the data-generating process of the trend and cycle components, which in general makes the choice of its optimal value almost impossible unless we know in detail what these generating processes are (Adams & Coe, 1990; Laxton & Tetlow, 1992 and Apel & Jansson, 1999). Finally, it is a well established fact in the literature (for example Beneš et al., 2010 and Melolinna & Tóth, 2016) that simple, univariate filters suffer from an "end-point" problem[3]. This property

[2]Early researchers used the methods that extracted the trend and cycle only from the information contained in the output series (without using any additional information from other series), hence the name univariate filters.

[3]With additional incoming data, estimates close to the end of the given sample usually get revised significantly.

makes decomposition approaches relying on the conventional univariate filters inappropriate for a real-time policy analysis[4].

An important class of alternatives to univariate dynamic methods is focused on the use of multivariate filters to estimate potential output (for example Laxton & Tetlow, 1992; Kuttner, 1994; Butler, 1996; Basistha & Startz, 2008; Beneš et al., 2010; Fleischman & Roberts, 2011; Morgan et al., 2019 and Tóth, 2019). The majority of aforementioned research apply a particularly useful class of models, termed structural time series models (STM) or UCM, which were first introduced into macroeconomics by Engle (1979), Watson and Engle (1983), Harvey and Todd (1983), Harvey (1985) and Watson (1986). In general, multivariate filtering approach introduces some economic structure to estimates by incorporating also information from the well known empirical relationships, such as Phillips curve relating the inflation and the output gap and Okun's law connecting the unemployment gap and the output gap (Cerra & Chaman Saxena, 2000). In that sense, estimates of the potential output and the output gap become consistent with Okun's (1962) definition of potential output (Apel & Jansson, 1999). An advantage of this approach is that its basic form is still relatively easy to implement, since it requires only a few variables and it can be relatively straightforwardly augmented, where data availability permits (Alichi, 2015). In addition, the estimates provided by these types of models may not deviate too much from actual data, which helps to capture shocks that may have lasting effects on the economy and lead to unexpected revisions of potential output. These features make the multivariate filtering techniques particularly useful for measuring potential output in the aftermath of the global financial crisis. On the other hand, the shortcomings of such an approach are similar to those already discussed in the case of univariate techniques, with two of them being particularly relevant: first, "end-point" puzzle remains an important problem and second, the more complicated methods bring improvement over the simple statistical filtering only if the structural relationships specified in the extended system are valid in the economy in question (Alichi et al., 2017).

Another common technique to estimating potential output is related to quasi-theoretical methods, more specifically to the production function approach (for example Denis et al., 2006; Beffy et al., 2006; Havik et al., 2014 and Turner et al., 2016). It provides a comprehensive economic framework for estimating potential output and in its simplest form it can be represented with a clear link between the output, the level of technology and the inputs of production (usually labour and capital) using a conventional Cobb-Douglas production function (Cobb & Douglas, 1928). The usual estimation procedure consists of obtaining data on employment and capital stock, and collecting

[4]Contrary to the conventional belief some authors see an advantage in this since it can represent an effective means of capturing structural breaks (for example Gerdrup et al., 2013)

total factor productivity (TFP) as the residual from the production function equation. The estimates of potential output are in the second step retrieved by combining smoothed estimate of the TFP series and process for "potential employment" (trend components) with the estimate of the capital stock. This approach allows us to closely examine the drivers of potential output growth, while its limitations are mainly related to the accessibility of reliable capital stock data and to the quality of filtering methods used to detrend TFP and employment components (Blagrave et al., 2015)[5].

In recent years some important alternative techniques gained popularity. First, applies the NK-DSGE models to estimate potential output and the output gap (for example Juillard et al., 2006 and Vetlov et al., 2011). These models are derived from microeconomic foundations i.e. optimizing agents, which usually form rational expectations and maximize their objective functions subject to their constraints, and therefore present theoretically the most rigorous approach. Furthermore, they are build on three crucial elements, which are not covered in the previously mentioned, more empirically oriented techniques. First, NK-DSGEs rely on the theory of optimal monetary policy, meaning that model based measures of potential output are consistent with the policy making decisions related to the output gaps. Second, they exploit advances in the estimation of NK-DSGE models, which allow a quantitative, internally consistent and fully structural interpretation of the macroeconomic fluctuations (especially dynamics in inflation, actual and potential output). Lastly, besides using model consistent concept of potential output, NK-DSGE structure allows utilizing also more traditional concepts of potential output that are, due to the general equilibrium setup, consistent with optimal monetary policy decisions (Vetlov et al. 2011). The second class of alternatives builds on extending the analysis to open economy framework or even including other important macroeconomic relations in the otherwise standard UCM approach. For example we can mention research by Alberola et al. (2013) who have expanded the definition of potential output to include also global imbalances, while another strand of literature is focusing more on including either domestic financial imbalances (for example Borio et al., 2013, 2014 and Melolinna & Tóth, 2016) or more broadly defined concepts like current account balance (for example Darvas & Simon, 2015) in the definition of potential output.

Regarding the already existing measures of potential output for Slovenia, provided by the international institutions, European Commission (EC) (Havik et al., 2014), Organisation for Economic Co-operation and Development (OECD) (Turner et al., 2016) and International Monetary Fund (IMF) (De Masi, 1997) regularly provide their own estimates by utilizing aggregate

[5]If the employment and TFP series are detrended using an H-P filter, then the resulting potential output estimates will relatively closely follow the estimates obtained form a direct application of H-P filter on GDP data.

production function methodology using trend input components. When it comes to econometric modelling in a multivariate filtering context, the current paper is one of the first to close the existing gap in Slovenian potential output and output gap literature. It focuses on the already mentioned second stream of alternative techniques and attempts to apply the combination of production function and UCM methodologies to the Slovenian potential output estimation study. In that sense results of the exercise reflect the most up to date findings, issues and tendencies related to that field.

Lastly, Table 1 summarizes the literature on modelling methodologies for potential output and output gap estimation.

Table 1: Summary of the modelling methodologies for potential output and output gap estimation

Modelling methodology	Research	Pros (+) vs. Cons (-)
Univariate filters	Hodrick and Prescott (1981, 1997) Beveridge and Nelson (1981) King and Rebelo (1993) Baxter and King (1999)	+) Simplicity. +) Coherency. -) No consideration of other relevant economic information and structures (i.e. estimates are considered more of a “trend” growth). -) Sensitivity to several statistical features (e.g. parameter λ). -) “End-point” problem.
Multivariate filters	Adams and Coe (1990) Laxton and Tetlow (1992) Kuttner (1994) Butler (1996) Apel and Jansson (1999) Basistha and Startz (2008) Beneš et al. (2010) Fleischman and Roberts (2011) Blagrave et al. (2015) Alichi (2015) Alichi et al. (2017) Morgan et al. (2019) Tóth (2019)	+) More economic structure. +) Relatively easily augmented. +) Able to capture shocks that have lasting effects on the economy. -) “End-point” problem. -) More complicated methods may not bring improvement (validity of specified structural relationships for the economy in question).
Quasi-theoretical methods (prod. function approach)	De Masi (1997) Denis et al. (2006) Beffy et al. (2006) Havik et al. (2014) Turner et al. (2016)	+) Able to closely examine the drivers of potential output growth. -) Accessibility of reliable capital stock data. -) Quality of filtering methods used to detrend TFP and employment components.
NK-DSGE	Juillard et al. (2006) Vetlov et al. (2011)	+) Theoretically the most rigorous approach. +) Reliance on the theory of optimal monetary policy (consistent estimates). +) Advanced estimation techniques allow fully structural interpretation of macroeconomic fluctuations. -) Complexity.
Multivariate filters (extensions)	Alberola et al. (2013) Borio et al. (2013, 2014) Darvas and Simon (2015) Melolinna and Tóth (2016)	+) Extension with global imbalances. +) Extension with domestic financial imbalances. +) Extension with current account balance. -) Please see multivariate filters.

Source: *Own specification.*

3 Modelling methodology

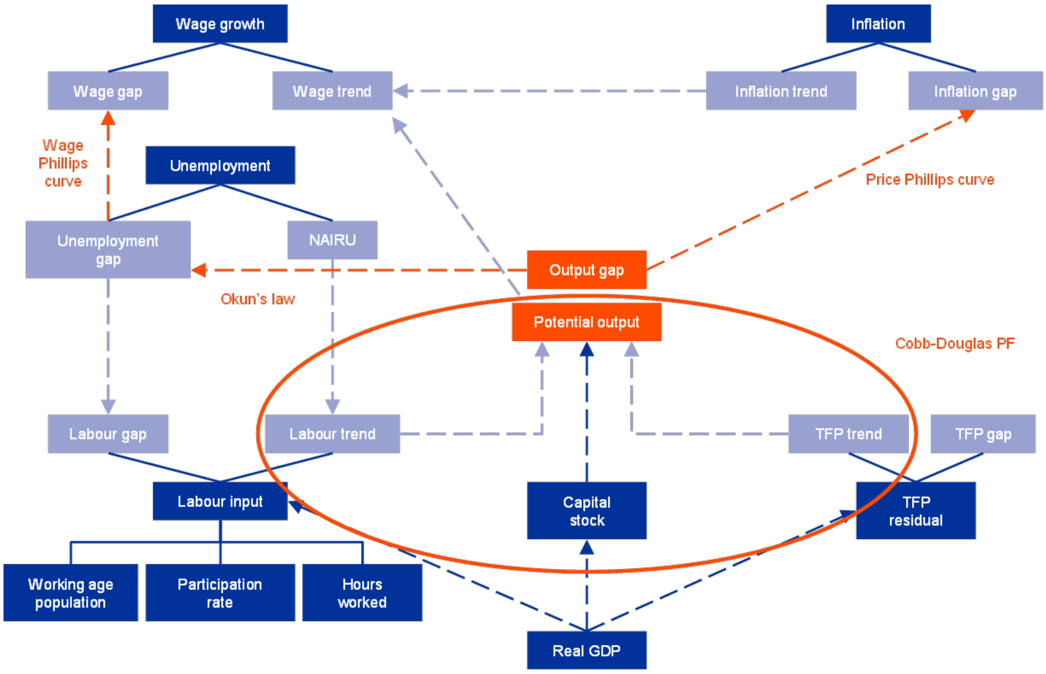
In the present paper, the combination of production function and UCM methodology is used, which closely follows the outcome of the internal work of the WGF Working Group Task Force on Potential Output[6] (Morgan et al., 2019 and Tóth, 2019). The applied multivariate filtering approach builds on the framework introduced by Laxton and Tetlow (1992) and Kuttner (1994) and is similar to the models utilized by Beneš et al. (2010), Blagrave et al. (2015), Melollina and Tóth (2016) and Alichi et al. (2017). Special feature of our model is the adoption of the combination of the two usually separated frameworks (production function approach and UCM), which gained some popularity among the central bankers in the recent years, simply due to its ability to incorporate more economic structure and consequently more relevant information in the potential output and output gap estimation process. This Section describes in detail the backbone state-space system that is developed for the selected approach.

3.1 Extended UCM framework

The main aim of the study is to develop a method that combines a small semi-structural UCM with a production function approach for the specific case of Slovenia. This combination allows potential output estimates to incorporate more economic structure than within traditional production function approach, while it still preserves the production function as a key organising element. Figure 1 depicts a stylised representation of the model.

[6]WGF stands for Working Group on Forecasting, which is one of the three working groups reporting to the Monetary Policy Committee (MPC) and is composed of European Central Bank's (ECB's) and euro area National Central Bank's (NCB's) experts. Its main responsibility is the preparation of detailed figures for the macroeconomic projections for the euro area and for the individual euro area countries (European Central Bank, 2016). In 2015, WGF ECB staff decided to set up a Working Group Task Force on Potential Output to consider a range of issues relating to potential output.

Figure 1: Stylised representation of the extended UCM



Source: *Andersson et al. (2018)*

The representation of the backward-looking model in a state-space form[7] allows us to utilize Kalman filtering techniques to decompose six key observable variables of our model (real GDP, unemployment rate, core inflation, wage inflation, labour force participation rate and hours worked per person) into trend and cyclical components. The model incorporates some well known empirical (macroeconomic) relationships which are reflected not only in the production function but also in wage and price Phillips curves and Okun's law. In addition, a number of auxiliary variables enter the model where some of them are simply included as exogenously determined observables (for example capital stock and working age population), while others are endogenously decomposed into trend and cycle, where only their trend components enter in the production function equation (for example labour force participation rate and average hours worked).

The measurement equations specified below (Equations 1 to 8) demonstrate how the 8 observable variables (left hand side variables) are linked to their unobservable counterparts (right hand side variables), where variables with hats denote cyclical components and variables with bars denote trend components. Observable variables are expressed mainly in logarithms or logarithmic differences (utilized for HICP excluding energy and compensation to employees -

[7]State-space representation of the extended UCM is shown in the Appendix A.

per head), where the only exception is unemployment rate, which is measured as percentage of the labour force.

$$\text{Output (real GDP)} \qquad y_t = \bar{y}_t + \hat{y}_t \tag{1}$$

$$\text{Unemployment rate} \qquad u_t = \bar{u}_t + \hat{u}_t \tag{2}$$

$$\text{Price inflation} \qquad \pi_t = \bar{\pi}_t + \hat{\pi}_t \tag{3}$$

$$\text{Wage inflation} \qquad w_t = \bar{w}_t + \hat{w}_t \tag{4}$$

$$\text{Labour force participation rate} \qquad lfpr_t = \overline{lfpr}_t + \widehat{lfpr}_t \tag{5}$$

$$\text{Average hours worked} \qquad ahw_t = \overline{ahw}_t + \widehat{ahw}_t \tag{6}$$

$$\text{Working age population} \qquad wap_t = \overline{wap}_t \tag{7}$$

$$\text{Capital stock} \qquad k_t = \bar{k}_t \tag{8}$$

The dynamic processes of the unobservable variables are presented by the transition equations which form the block of state equations (Equations 9 to 17). The trend-cycle decomposition of output (real GDP) is based on a local linear trend structure, where the output gap follows an $AR(2)$ process and the trend output is modelled as a Cobb-Douglas production function. Regarding the specific inputs to the trend output, working age population and capital stock enter the production function exogenously (i.e. their trend measures are equal to their observed values), while the NAIRU (or trend unemployment rate), the trend participation rate, the trend average hours worked and the trend TFP are all endogenously driven. In particular, the trend participation rate and trend average hours worked are filtered using the state-space formulation of the H-P filter, where the implied smoothness parameter (λ) resembles the squared ratio of cyclical (hat) and trend shifter (tilde) shocks. Lastly, trend TFP is modelled as an integrated process of order 1 i.e. $I(1)$.

$$\text{Output gap} \qquad \hat{y}_t = \alpha_1 \hat{y}_{t-1} - \alpha_2 \hat{y}_{t-2} + \varepsilon_t^{\hat{y}} \tag{9}$$

$$\text{Output trend} \qquad \begin{aligned} \bar{y}_t = \bar{y}_{t-1} + \Delta\overline{tfp}_t + \iota\big[\Delta\overline{wap}_t + \Delta\overline{lfpr}_t + \Delta\overline{ahw}_t + \\ + \Delta\ln(1-\bar{u}_t)\big] + (1-\iota)\,\Delta\bar{k}_t \end{aligned} \tag{10}$$

$$\text{TFP trend growth rate} \qquad \Delta\overline{tfp}_t = \Delta\overline{tfp}_{t-1} + \varepsilon_t^{\Delta\overline{tfp}} \tag{11}$$

$$\text{Capital stock trend} \qquad \begin{aligned} \bar{k}_t &= \bar{k}_{t-1} + \tilde{k}_t \\ \tilde{k}_t &= \tilde{k}_{t-1} + \varepsilon_t^{\tilde{k}} \end{aligned} \tag{12}$$

$$\text{Working age population trend} \qquad \begin{aligned} \overline{wap}_t &= \overline{wap}_{t-1} + \widetilde{wap}_t \\ \widetilde{wap}_t &= \widetilde{wap}_{t-1} + \varepsilon_t^{\widetilde{wap}} \end{aligned} \tag{13}$$

$$\text{Participation rate gap} \qquad \widehat{lfpr}_t = \varepsilon_t^{\widehat{lfpr}} \tag{14}$$

$$\text{Participation rate trend} \qquad \begin{aligned} \overline{lfpr}_t &= \overline{lfpr}_{t-1} + \widetilde{lfpr}_t \\ \widetilde{lfpr}_t &= \widetilde{lfpr}_{t-1} + \varepsilon_t^{\widetilde{lfpr}} \end{aligned} \tag{15}$$

$$\text{Average hours gap} \qquad \widehat{ahw}_t = \varepsilon_t^{\widehat{ahw}} \tag{16}$$

$$\text{Average hours trend} \qquad \begin{aligned} \overline{ahw}_t &= \overline{ahw}_{t-1} + \widetilde{ahw}_t \\ \widetilde{ahw}_t &= \widetilde{ahw}_{t-1} + \varepsilon_t^{\widetilde{ahw}} \end{aligned} \tag{17}$$

We devote special attention to the last three sub-blocks of state equations, as they impose some additional economic structure into an otherwise mostly mechanical state-space system. First, the unemployment rate is decomposed into trend and cyclical components (Equations 18 and 19), where the former is connected to the output gap via an Okun's law relationship, while the latter (the NAIRU) follows an $I(1)$ process, with an $AR(1)$ process governing its growth rate[8].

$$\text{Okun's law} \qquad \hat{u}_t = \gamma_1 \hat{u}_{t-1} - \gamma_2 \hat{y}_{t-1} + \varepsilon_t^{\hat{u}} \tag{18}$$

$$\text{NAIRU} \qquad \begin{aligned} \bar{u}_t &= \bar{u}_{t-1} + \tilde{u}_t \\ \tilde{u}_t &= \kappa \tilde{u}_{t-1} + \varepsilon_t^{\tilde{u}} \end{aligned} \tag{19}$$

[8]It can optionally also take into account the changes in the long-term unemployment rate, but we decided to exclude that channel, as the unavailability of data after 2018Q4 makes the long-term unemployment rate rather uninformative to any trend unemployment rate variations in the long-term horizon.

Second, similarly to unemployment rate breakdown, also inflation (HICP excluding energy[9]) is decomposed into cyclical and trend components (Equations 20 and 21). A cyclical inflation (specified by price Phillips curve) relates the inflation gap to the output gap, while trend inflation is assumed to follow an $AR(1)$ process anchored by an ECB's inflation target (π^*).

$$\text{Price Phillips curve} \qquad \hat{\pi}_t = \beta_1 \hat{\pi}_{t-1} + \beta_2 \hat{y}_{t-1} + \varepsilon_t^{\hat{\pi}} \tag{20}$$

$$\text{Trend inflation} \qquad \bar{\pi}_t = (1-\varphi)\pi^* + \varphi \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}} \tag{21}$$

Third decomposition is associated with the growth in wages (compensation per employee) (Equations 22 and 23). A cyclical part of the decomposed observed series (wage Phillips curve) connects the wage inflation gap to the unemployment gap, while trend wage inflation is modelled as the sum of trend inflation and trend labour productivity growth (trend output divided by trend employment in persons) in order to capture the long-run relationship among included variables. In addition, trend labour productivity term in Equation 23 is multiplied by 4 as both wage and price inflation are defined in annual terms.

$$\text{Wage Phillips curve} \qquad \hat{w}_t = \beta_3 \hat{w}_{t-1} - \beta_4 \hat{u}_{t-1} + \varepsilon_t^{\hat{w}} \tag{22}$$

$$\text{Trend wage inflation} \qquad \begin{aligned} \bar{w}_t = \bar{\pi}_t + 4 \cdot \big\{ \Delta \bar{y}_t - \big[\Delta \overline{wap}_t + \Delta \overline{lfpr}_t + \\ + \Delta \ln (1 - \bar{u}_t) \big] \big\} + \varepsilon_t^{\bar{w}} \end{aligned} \tag{23}$$

At the end, it should be stressed that "end-point" problem does not necessarily vanish when using more sophisticated and complex trend-cycle decomposition procedures, such as the combination of production function and UCM-based methodology. Nevertheless, more complex methods usually perform better, since they exploit the information content of variables, which, according to the established economic theory, tend to co-move with the dynamics observed in the output (Orphanides & van Norden, 2002; Melollina & Tóth, 2016; Morgan et al., 2019 and Tóth, 2019). More detailed discussion on this issue (i.e. the comparison between revisions of the output gap

[9]Slovenian HICP inflation has shown to be sensitive to the spillovers of foreign prices. Although the literature recognizes oil and food prices as main external factors influencing domestic consumer prices (Ciccarelli & Osbat, 2017 and Parker, 2017), fluctuations in food prices (which are mainly related to volatile prices in fruits and vegetables i.e. unprocessed food) have in history influenced Slovenian HICP inflation dynamics in much smaller extent than developments in oil prices and international energy prices (Bank of Slovenia, 2017). This can be partly related to the smaller weight of unprocessed food in Slovenian HICP inflation (from 2007 onwards, average weight for unprocessed food was around 6.7%, while in the case of energy prices it was around 13.4%).

estimates produced by the H-P trend-cycle decomposition and the extended UCM approach) is however postponed to the second part of the next Section.

4 Empirical analysis

This section of the paper presents the empirical analysis and its main findings. It is divided into two Subsections: the first part presents the data set for Slovenian economy and all the necessary statistical techniques and data transformations applied. Furthermore, the Bayesian estimation technique utilized for parameter estimation is described in detail. In the second part, main results of the empirical exercise done with the extended UCM are presented and discussed[10].

4.1 Data and estimation methodology

The data set used in the study consists of 9 seasonally adjusted quarterly time series that are also used in the regular BMPE projection exercise[11]. Most of the original series for relevant aggregates of Slovenian economy are obtained from the Statistical office of the Republic of Slovenia (SORS)[12] i.e. real GDP (y_t), compensation to employees - per head (w_t), unemployment rate (u_t), total labour force (lfn_t) (after 2008Q1), whole economy employment - heads (lnn_t) and whole economy employment - average hours worked (lhn_t), while data on HICP excluding energy (π_t), total labour force (lfn_t) (before 2008Q1) and working age population (wap_t) are retrieved from Eurostat. In addition, historical data for the real capital stock series (k_t) is calculated internally, using the perpetual inventory method[13], while labour force participation rate ($lfpr_t$) and average hours worked (ahw_t) are calculated using the following standard formulas:

$$\text{Labour force participation rate} \qquad lfpr_t = \frac{lfn_t}{wap_t} \tag{24}$$

$$\text{Average hours worked} \qquad ahw_t = \frac{lhn_t}{lnn_t} \tag{25}$$

[10]The model code is implemented in MATLAB using IRIS Toolbox (Beneš et al., 2015). The basic MATLAB code, on which the empirical analysis builds on, has been developed by Máté Tóth (mate.toth@ecb.int), ECB, Directorate General Economics, Output and Demand division. Specific changes for the Slovenian case were done by the author of the paper.

[11]BMPE stands for Broad Macroeconomic Projection Exercise, which is conducted twice a year (in June and December) within the framework of the ECB forecasting process and involves staff members from both the euro area NCBs and the ECB (European Central Bank, 2016). Series are seasonally adjusted using the X-12 method.

[12]Text in the brackets provides a direct link to the model variable represented in Section 3.

[13]For more detailed treatment of the perpetual inventory method for calculating real capital stock series see Jemec (2012).

The entire sample of the empirical exercise ranges from 1995Q1 to 2018Q4, while, before the estimation process, each individual series is also transformed using one of the following three methods[14]:

- Rates $\Rightarrow y_t = \dfrac{Y_t}{100}$ (applied to u_t);
- Annualised quarter-on-quarter changes $\Rightarrow y_t = 4\times(1-L)\log(Y_t)$ (applied to $\log(\pi_t)$ and $\log(w_t)$);
- Logarithms $\Rightarrow y_t = \log(Y_t)$ (applied to y_t, π_t, w_t, lfn_t, wap_t, lhn_t, lnn_t, k_t, $lfpr_t$ and ahw_t);

Due to the different length and the availability of data, the estimation sample, used for parameter estimation, ranges from 1996Q1 to 2018Q4 (most of the data are available from 1996Q1), while the smoothing exercise is conducted over an estimation sample which is extended until 2021Q4 (in line with projection horizon in June 2019 BMPE projections exercise).

In order to be able to obtain time paths of unobservable variables, we express the extended UCM in the state-space form and estimate its parameters. The usual approach applies Kalman filter to evaluate the log-likelihood function of the model, which, in principle, produces maximum likelihood estimates of the parameters. However, the immediate problem that can arise is the "curse of dimensionality"[15], which in many cases leads to a poorly identified regions of the parameter space and unreasonable parameter values. The most convenient way to overcome the aforementioned difficulty is to apply Bayesian techniques, which implicitly shrink the likelihood surface and make estimation of the parameters feasible. In order to conduct Bayesian estimation, we combine the prior assumptions (prior distributions of parameters for specific case of Slovenia are needed) and the information content in the data to obtain the posterior distribution of the parameters.

[14] The transformation of the variables correspond to the units of the y-axes in Figure B.1 in the Appendix B.

[15] This problem usually arises due to short data sample and unobservable nature of key variables of interest (Pelagatti, 2015 and Melolinna & Tóth, 2016).

Table 2: Prior and posterior parameter values

Parameter	Chart labels[16]	Prior density type	Hyper-parameters	Posterior median
α_1	alpha1	Gamma	[μ=1.5, σ=0.15]	1.4957
α_2	alpha2	Gamma	[μ=0.6, σ=0.15]	0.5996
ι	iota	Beta	[μ=0.67, σ=0.01]	0.6711
β_1	beta1	Beta	[μ=0.7, σ=0.15]	0.4231
β_2	beta2	Gamma	[μ=0.5, σ=0.15]	0.3250
φ	phi	Beta	[μ=0.7, σ=0.15]	0.9965
β_3	beta3	Beta	[μ=0.7, σ=0.15]	0.6723
β_4	beta4	Gamma	[μ=0.5, σ=0.15]	0.3465
γ_1	gamma1	Beta	[μ=0.7, σ=0.15]	0.7574
γ_2	gamma2	Gamma	[μ=0.5, σ=0.15]	0.2012
κ	kappa	Gamma	[μ=0.7, σ=0.15]	0.5744
$\varepsilon_t^{\hat{y}}$	std_eps_y_hat	Inverse gamma	[μ=1, σ=∞]	0.0127
$\varepsilon_t^{\Delta\overline{tfp}}$	std_eps_tfp_bar	Inverse gamma	[μ=0.01, σ=∞]	0.0015
$\varepsilon_t^{\hat{\pi}}$	std_eps_pi_hat	Inverse gamma	[μ=1, σ=∞]	0.0218
$\varepsilon_t^{\bar{\pi}}$	std_eps_pi_bar	Inverse gamma	[μ=0.01, σ=∞]	0.0016
$\varepsilon_t^{\hat{w}}$	std_eps_w_hat	Inverse gamma	[μ=1, σ=∞]	0.0209
$\varepsilon_t^{\bar{w}}$	std_eps_w_bar	Inverse gamma	[μ=0.01, σ=∞]	0.0028
$\varepsilon_t^{\hat{u}}$	std_eps_u_hat	Inverse gamma	[μ=1, σ=∞]	0.0121
$\varepsilon_t^{\tilde{u}}$	std_eps_u_tilde	Inverse gamma	[μ=0.01, σ=∞]	0.0015
$\varepsilon_t^{\widehat{lfpr}}$	std_eps_lfpr_hat	Inverse gamma	[μ=1, σ=∞]	0.0210
$\varepsilon_t^{\widetilde{lfpr}}$	std_eps_lfpr_tilde	Inverse gamma	[μ=0.01, σ=∞]	0.0014
$\varepsilon_t^{\widehat{ahw}}$	std_eps_ahw_hat	Inverse gamma	[μ=1, σ=∞]	0.0175
$\varepsilon_t^{\widetilde{ahw}}$	std_eps_ahw_tilde	Inverse gamma	[μ=0.01, σ=∞]	0.0014
$\varepsilon_t^{\widetilde{wap}}$	std_eps_wap_tilde	Inverse gamma	[μ=0.01, σ=0.01]	0.0042
$\varepsilon_t^{\tilde{k}}$	std_eps_k_tilde	Inverse gamma	[μ=0.01, σ=0.01]	0.0009

Source: *Own calculations.*

Table 2 presents the current parametrisation of the model. Similarly as in Melolinna and Tóth (2016), Morgan et al. (2019) and Tóth (2019), the specified parameter values reflect that the cyclical components of the model, which follow an $AR(1)$ processes, are quite persistent, while the main driver behind fluctuations in observable variables are assumed to be cyclical rather than trend shocks. The theoretical foundations behind Phillips curve and Okun's law are reflected in the role of output gap in the cyclical inflation and unemployment gap equations. All the aforementioned characteristics of the parameter values result in Beta type prior distribution of $AR(1)$ parameters with mean 0.7

[16] Chart labels correspond to the titles of the posterior distribution charts (Figure B.4), provided in the Appendix B.

and standard deviation 0.15, while for other coefficients we assume Gamma type prior distributions with mean 0.5 and standard deviation 0.15. By using the specific type of prior distributions with corresponding hyper-parameters, we introduce macroeconomic theory (restrictions) in the process of estimation. Regarding the cyclical (trend) shock parameters, we assume the Inverse gamma type prior distribution with mean 1 (0.01 respectively) and standard deviation ∞. The production function parameter ι and the shock parameters $\varepsilon_t^{\widetilde{wap}}$ and $\varepsilon_t^{\tilde{k}}$ are the only ones that cannot be identified from the data. Therefore, they are calibrated to 0.67 (corresponds to the average labour share historically observed in Slovenia), 0.01 and 0.01, respectively[17].

With regard to the process of simulation, posterior medians are found via a numerical optimization of the combined log-prior and log-likelihood function using the particle swarm optimisation algorithm (Kennedy & Eberhart, 1995 and Shi & Eberhart, 1998), while posterior distributions are generated via Markov Chain Monte Carlo (MCMC) simulations based on adaptive random walk Metropolis posterior simulator with 3,000,000 draws and a 50% burnin (Robert & Casella, 2004; Gelman et al., 2014 and Sariola, 2019).

The model structure and its implementation in MATLAB through the IRIS Toolbox (Beneš et al., 2015) also allows for straightforward introduction of expert judgement. For the specific case of Slovenia, an additional variable (observations) was added to the model and linked directly to trend inflation (unobservable) due to strong disinflation at the beginning of the sample, which was predominantly related to the processes of joining the European Exchange Rate Mechanism II (ERM II) and later the euro area[18]. Doing that we informed the model that the early high inflation period should not necessarily

[17]To be more precise, we impose a tight prior on all three parameters by setting their standard deviations to 0.01.

[18]In line with the recent debate on the dynamics of trend inflation in the euro area and its countries (Ciccarelli & Osbat, 2017 and Rostagno et al., 2019), the stochastic trend assumption over the entire sample was also considered. This was implemented by analysing two alternative specifications of the Equation 21:

$$\text{Trend inflation} \qquad \begin{aligned} \bar{\pi}_t &= \bar{\pi}_{t-1} + \tilde{\pi}_t \\ \tilde{\pi}_t &= \tilde{\pi}_{t-1} + \varepsilon_t^{\tilde{\pi}} \end{aligned}$$

and

$$\text{Trend inflation} \qquad \bar{\pi}_t = \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}}$$

Nevertheless, in both cases the forecasting performance of the extended UCM in comparison to 4-variable (B)VAR models decreased significantly, especially when considering the accuracy of core and wage inflation projections.

be interpreted as overheating. The additional equation needed for this is specified as:

$$\bar{\pi}_t^{obs} = \bar{\pi}_t + \varepsilon_t^{\bar{\pi}^{obs}} \tag{26}$$

Table 3: Extended UCM specification

Specification item	Modification
Estimation sample	1996Q1-2018Q4
Filtering sample	1996Q1-2021Q4
Non-BMPE variables	None.
Other data related issues	Inflation is measured by HEX (HICP excl. energy), instead of HEF (HICP excl. food and energy).
Equation blocks switched on	$LTU = 0$, $PPC = 1$, $WPC = 1$, $TFPCAPU = 0$
Modifications to equations	$LAN\_$ and $K\_$ are not treated as an $I(1)$ processes, but are represented as an $I(2)$ processes.
Modification to the estimation/ filtering procedure	1) In the estimation, tight prior on labour share in Cobb Douglas production function (ι) and the shock parameters ($\varepsilon_t^{\widetilde{wap}}$ and $\varepsilon_t^{\tilde{k}}$) are used. In addition, prior value of ι is modified to be more in line with Slovene economy. 2) Prior values of gap parameters (β_1, β_3 and γ_1) and trend unemployment parameter (κ) are set to 0.7, prior variance of parameters is reduced in order to better target parameter values. 3) In 1995Q1-2006Q4, judgement on inflation is introduced via the H-P filter (as $PIE\_BAR\_$) due to strong disinflation in that period. 4) Number of draws in MCMC simulation: 3,000,000, burnin: 1,500,000 (50% of number of draws).

Source: *Own specification.*

Lastly, Table 3 summarizes the parts of the model and the parametrization that are, in comparison to the basic UCM developed by Morgan et al. (2019) and Tóth (2019), adjusted to the specific case of Slovenia.

4.2 Results of empirical exercise

The last Subsection presents and discusses the results of the empirical exercise undertaken with the extended UCM[19]. In the first part, Kalman filter and

[19] All additional figures not presented in the main text are available in the Appendix B.

Kalman smoother estimates of potential output, output gap and unemployment gap are compared in order to examine the quality and the validity of the system developed in Section 3. In addition, we present also NAIRU estimates, which reflect the evolution of the trend component of the unemployment rate. Next, potential output decomposition to unobservable variables (trend components) is presented and accompanied by the economic intuition behind the estimates. Furthermore, also long-term analysis of potential output developments is demonstrated by utilizing a set of long-term assumptions, which are mainly based on historical developments of included series (except for the labour market variables). Lastly, we present some results from the pseudo real-time analysis.

Figure 2 compares the results of Kalman filter (one-sided filter) and Kalman smoother (two-sided filter) estimates of output gap and unemployment gap that are obtained from the model[20]. The main difference between the two approaches is that, conditioned on observing all currently available information (i.e. all past and current observations), the filter updates the current value of unobservable components (state variables), while on the other hand the smoother shows how to infer value of unobservable components (state variables) for each period given the entire dataset (i.e. all past and future

[20] When considering a state-space system presented in Appendix A, the paths of unobservable components X_t can be explored by obtaining and comparing real time estimates $X_{t|t}$ and smoothed estimates $X_{t|T}$. The former can be utilized by applying the Kalman filter for each $t = 1, 2, \ldots, T$. Thus, at the beginning of period t we have the estimated value of previous period ($X_{t-1|t-1}$), which is based on the history of observations $Z_{t-1}, Z_{t-2}, \ldots, Z_0$ and has some covariance matrix ($P_{t-1|t-1}$). As prior information, we also have Equation A.1, so that we can forecast a value conditional on information at period $t-1$ in the following way:

$$X_{t|t-1} = AX_{t-1|t-1}$$

New information related to X_t arrives in period t in the form of Z_t according to Equation A.2. As a result, estimates of X_t are updated by combining the two sources of information in the following way:

$$X_{t|t} = X_{t|t-1} + K_t\big(Z_t - \underbrace{DX_{t|t-1}}_{Z_{t|t-1}}\big)$$

where the term $Z_t - Z_{t|t-1}$ is the innovation and K_t is the Kalman gain. In each iteration we also store covariance matrices ($P_{t|t-1}$ and $P_{t|t}$). Given the sequence $\{X_{t|t-1}, X_{t|t}, P_{t|t-1}, P_{t|t}\}_{t=1}^{T}$, Kalman smoother on the other hand allows to infer the value of X_t for each $t = T-1, T-2, \ldots, 1$ given the entire dataset $Z_T = \{Z_1, Z_2, \ldots, Z_T\}$ i.e. $X_{t|T}$. In order to describe this procedure, we focus our attention on the Equation A.1. We start the smoothing with the last filtered observation $X_{T|T}$ and consider the following updating equation:

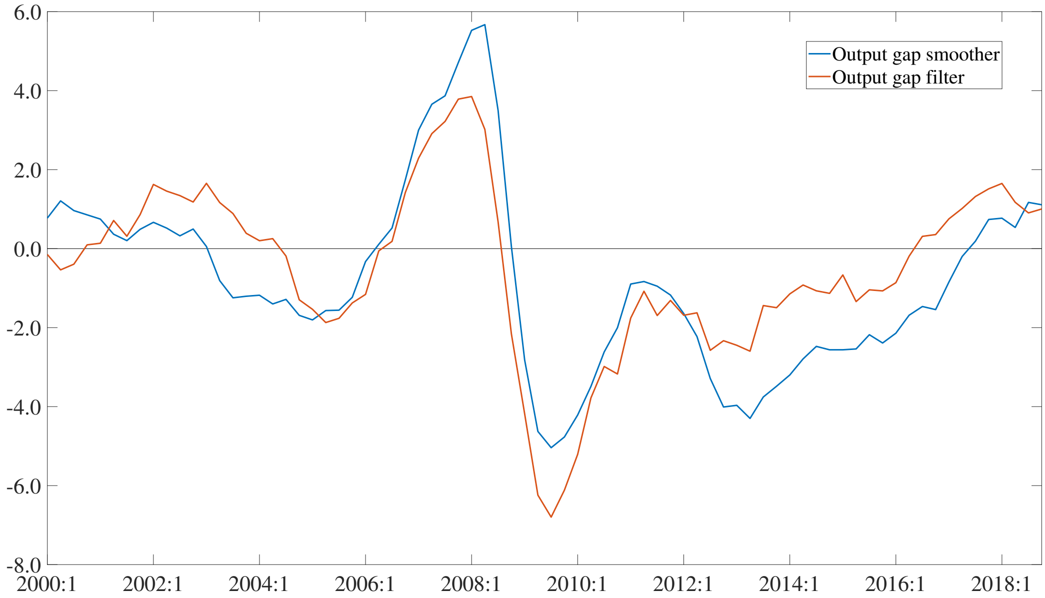
$$X_{t|T} = X_{t|t} + J_t\left(X_{t+1} - X_{t+1|t}\right)$$

This shows that the smoothed value $X_{t|T}$ is a function of the filtered (real time) value $X_{t|t}$ and the innovation on X in the next period $X_{t+1} - X_{t+1|t}$ (Hamilton, 1994; Kim & Nelson, 1999).

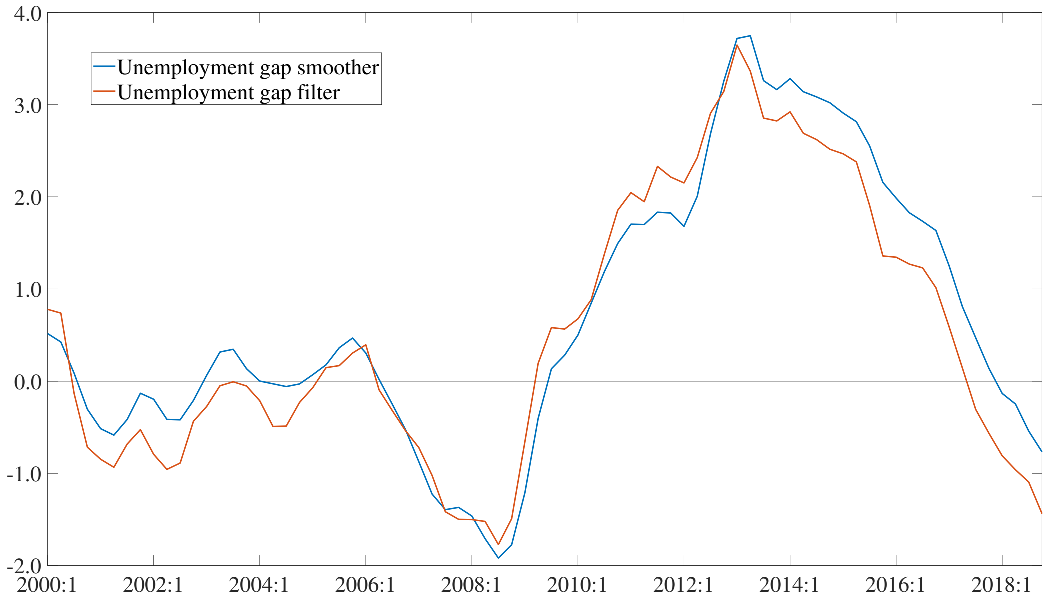
observations). The small difference between the two obtained series therefore implies that underlying filter estimates of extended UCM are able to produce economically sound potential output and output gap estimates by adequately taking into account information coming from the model.

Figure 2: Smoother vs. filter estimates

(a) Output gap (in % of potential output)



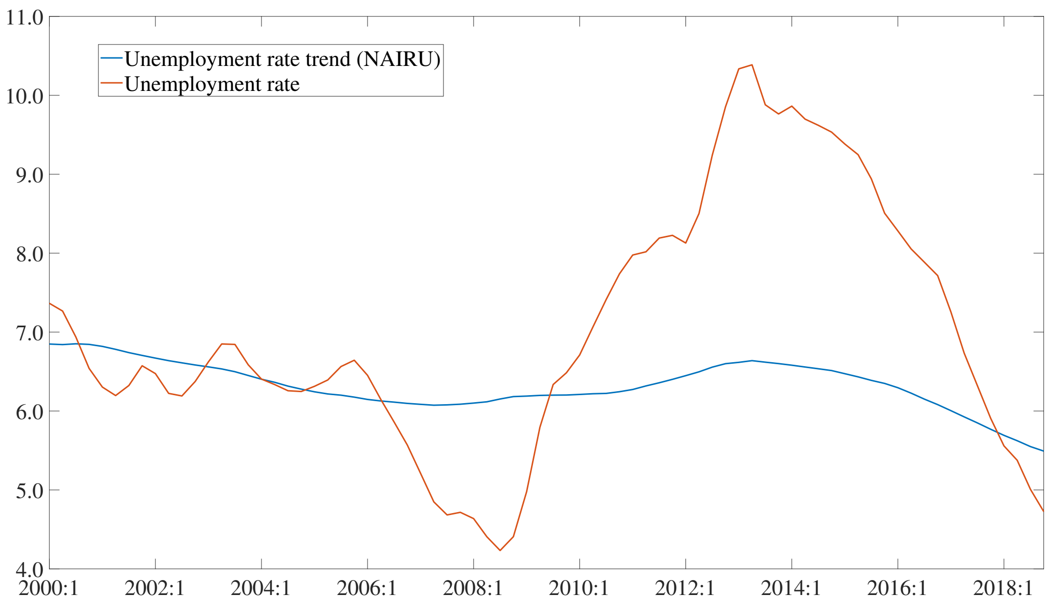
(b) Unemployment gap (diff. between unemployment rate and NAIRU in %)



Source: *Own calculations.*

Based on both definitions, we can see that filter estimates (updated estimates of the model) are able to adequately replicate pre-crisis fluctuations, as the estimates detect a relatively large and persistent positive output gap in years 2007 and 2009. At the same time, filter estimates are able to capture the marked slowdown during the crisis years and quite successfully indicate the second recession wave (slump in activity due to sovereign debt crisis in 2013). Regarding the post-crisis developments, extended UCM filter estimates somewhat struggle to capture exact dynamics of the smoother series, as the trend path (i.e. level of potential output) during the crisis period has been significantly altered by the adverse dynamics of the macroeconomic variables in the model at that time[21]. Furthermore, the gap between the filter and the smoother estimates may be also explained by the lack of financial indicators in the extended UCM, as post-crisis gradual improvement in financing conditions, which partially contributed to restoration of production capacities that collapsed in the crisis period, may have proved to be decisive for post-crisis potential output dynamics (Borio et al. 2013, 2014 and Melolinna & Tóth, 2016). Similar reasoning can be used to interpret the unemployment gap estimates in which the post-crisis difference between the filter and smoother estimates is somewhat smaller in comparison to output gap equivalent.

Figure 3: Unemployment rate and NAIRU (in %)



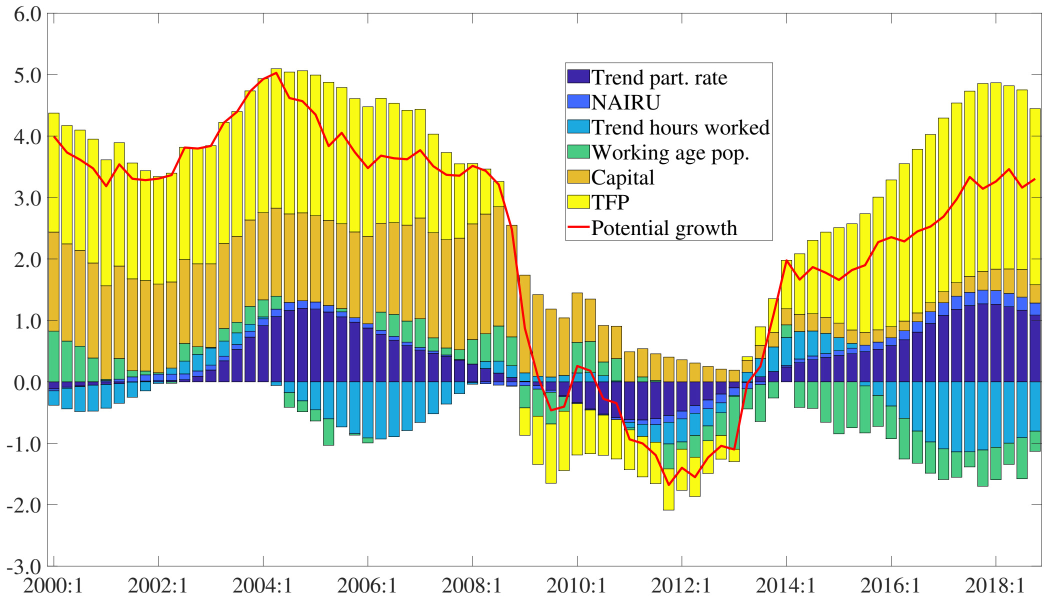
Note: *Realization (orange line) might deviate from the official seasonally adjusted series due to own seasonal adjustment method.*
Source: *SORS, Eurostat, Bank of Slovenia, own calculations.*

Figure 3 compares the actual unemployment rate series with its estimated

[21] In most of the post 2008 period there has also been a lack of inflationary pressures.

trend component i.e. NAIRU, which follows $I(1)$ process. From 2000 and until the beginning of 2008 extended UCM NAIRU estimates reflect declining path, which is in line with the pre-crisis developments, marked by the decrease in unemployment rate and gradual tightening on the labour market (i.e. deepening of the gap between supply and demand). In the period from 2008 to 2013, large increases in the unemployment rate and the downsizing of some sectors lead to an increase in NAIRU towards 7% therefore reflecting unfavourable developments on the labour market. Since 2013 both the registered unemployment rate and survey unemployment rate (ILO) have been decreasing and by that significantly influenced the path of the NAIRU. Post-crisis developments of the NAIRU are therefore a result of cyclical factors such as gradual improvement in general economic conditions and consequently progressive positive dynamics observed on the labour market.

Figure 4: Contributions to historical development of potential output growth (contributions in pp, y-o-y potential output growth in %)



Source: *Own calculations.*

Turning to the unobservable drivers (trend components) of potential output growth, Figure 4 shows its decomposition into contributions accounted for by TFP, capital and labour. The model specification of trend TFP follows an $I(1)$ process and on average represents the leading source of potential output growth over the entire horizon. Nevertheless, it has to be mentioned that changes of capital are endogenous to technological change, meaning that the contribution of technology in growth accounting exercise like this usually underestimates the full effect of technological change on output (Barro and Sala-i-Martin, 2004). As it can be observed in the Figure 4, the crisis had a notable negative impact on trend TFP growth, as it significantly affected long-term technological capacities (technology growth and efficiency were significantly influenced), with more marked slowdown observed already in years 2007 and 2008[22]. After the crisis, the improvement in TFP contribution can be attributable to gradual and broad based economic recovery as well as post-crisis restoration of production capacities of Slovenian firms (recent TFP developments can be also partly motivated by companies' incentives to adopt new technologies (Bank of Slovenia, 2019b)).

A significant contributor to the overall potential output growth was also capital. Except from the crisis period, we can see that capital and TFP contributions co-move, which is, as already stated, an indication of mutual relationship between the two and by that also a signal whether the investment activity is effectively increasing the production potential of the economy or not. Similarly as with TFP, we can observe a significant drop of capital contribution in the crisis years, which coincided with the collapse of investment activity. After the crisis, capital contribution and investment activity remained depressed for a longer period of time, mainly as a result of post-crisis deleveraging process, reconstruction of business models and significantly impacted risk profiles of the firms. In addition, despite high level of retained earnings and favourable financial conditions (due to accommodative monetary policy) in the recent years, demand for bank credit by Slovenian firms is still modest (Bank of Slovenia, 2019a), resulting in more gradual restoration of production capacities.

Regarding labour contribution, a more detailed decomposition to subcomponents reveals the labour market dynamics over the observed period. Before the crisis all the components were behaving pro-cyclically and were in line with the positive developments in the labour market, which is on the one hand reflected in the positive contributions of the trend participation rate, NAIRU and working age population and on the other hand in the negative contribution of the trend hours worked. The crisis period, marked by large increase in the unemployment rate and the severe cut-down of workforce in some sectors, is captured by negative contribution of all components to overall labour. After the downturn caused by crisis, the contributions of trend participation rate, NAIRU and trend hours worked started to recover, mainly on the back of gradual improvement in the labour market conditions. On the other hand, the major post-crisis drag can be observed in the contribution of the working age population, which has been shrinking at an increasing rate since 2011 and was mainly driven by demographical issues (population ageing). Only recently, these structural imbalances on the labour market are partly addressed by increasing hiring of foreign workers (Bank of Slovenia, 2019b).

[22]This may be partly explained by the increase of unproductive investment during the investment bubble at that time (European Commission, 2012).

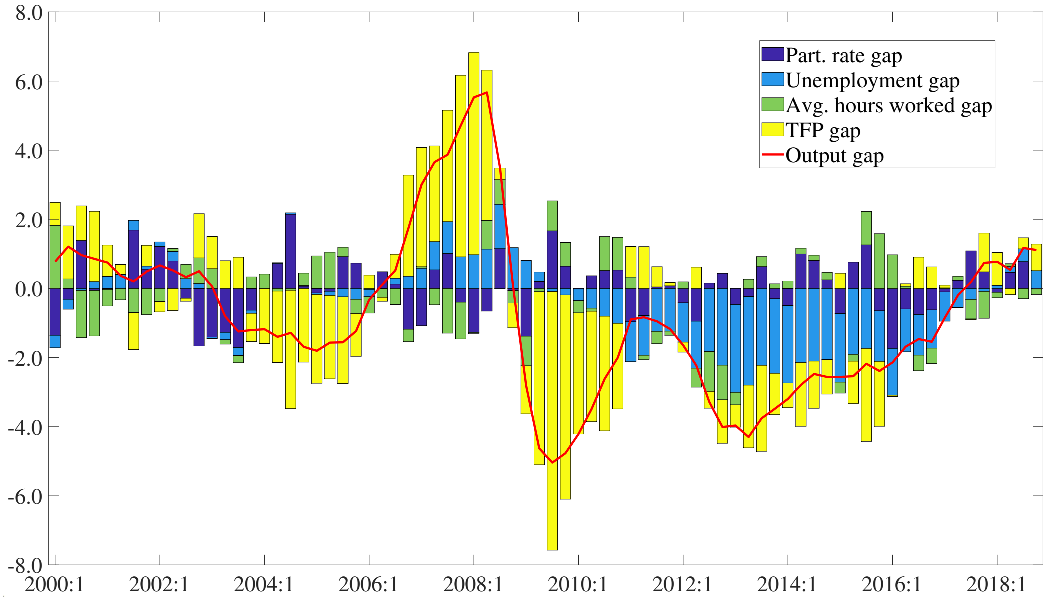
Table 4: Comparison of potential output and output gap estimates for Slovenia (potential output and NAIRU in %, output gap in % of potential output)

Variable	Estimate/institution	1999-2007	2008-2013	2014-2018
Potential output	Extended UCM	3.8	0.1	2.5
	EC	3.6	1.2	1.5
	OECD	3.4	1.3	1.9
	IMF	NA	NA	NA
	Average	3.6	0.9	2.0
Output gap	Extended UCM	0.3	-1.9	-1.2
	EC	1.7	-2.5	-1.0
	OECD	0.7	-1.8	-3.4
	IMF	0.9	-1.3	-2.1
	Average	0.9	-1.8	-1.9
NAIRU	Extended UCM	6.5	6.3	6.1
	EC	6.4	6.4	6.3
	OECD	6.3	6.3	6.9
	IMF	NA	NA	NA
	Average	6.4	6.3	6.4

Source: *EC, OECD, IMF, own calculations.*

In addition to the above decomposition, Table 4 summarizes the main results for Slovenia using the extended UCM and compares these to the Spring (Summer) 2019 estimates of the EC, the OECD and the IMF. Regarding the potential output figures, the estimates before the last financial crisis (1999-2007) reflect similar developments in potential growth among the institutions (around 3.5%), while in the later two periods extended UCM figures either mirror larger slump (2008-2013) or faster recovery of potential growth (2014-2018), which is also generally confirmed by the output gap estimates. Observed differences can be potentially explained by utilization of different methodologies, as extended UCM relies on combination of small semi-structural modelling and production function methodology and by that incorporates additional economic structure, unavailable within traditional production function approach. Extended UCM estimates therefore suggest that also other variables, included in the multivariate system, importantly influenced developments in potential growth and output gap in both aforementioned periods. On the other hand, NAIRU estimates reflect broadly comparable developments among the institutions in all three considered periods.

Figure 5: Contributions of unobservable gap variables to output gap (contributions in pp, output gap in % of potential output)

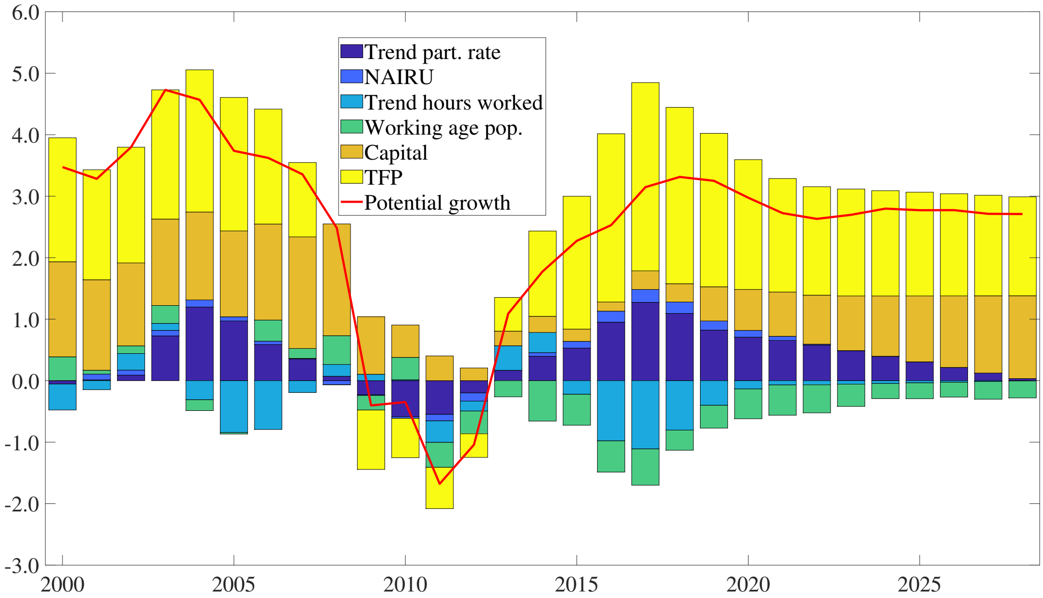


Source: *Own calculations.*

Figure 5 depicts a result from the decomposition of the smoother estimate of the output gap into contributions from the different unobservable gap variables included in the state-space system. As expected, the main driver of the pre-crisis fluctuations is the TFP gap (i.e. difference between TFP and trend TFP), which is generating a relatively large and persistent positive output gap before the last financial crisis. At the same time, negative developments in the TFP gap also represent a subcomponent that is driving the marked slowdown during the crisis years. Regarding the second recession wave, a somewhat different reasoning can be used as the economic slowdown is in this case more significantly influenced by unfavourable developments on the labour market which is further reflected in highly persistent negative contribution of unemployment gap (i.e. difference between unemployment rate and NAIRU). Most recent figures reflect gradual post-crisis recovery in both aforementioned subcomponents[23].

[23] As a matter of interest, Figures B.5 and B.6 in the Appendix B show two different measures of uncertainty around the output gap estimates.

Figure 6: Contributions to long-term potential output growth using the set of purely technical long-term assumptions (contributions in pp, long-term potential output growth in %)



Source: *Own calculations.*

Turning to the long-term developments, Figure 6 shows the results of the long-term forecasting exercise that reflect economically sound long-term narrative. Results of the simulation are obtained using a set of purely technical long-term assumptions which can be condensed in the following five points[24]:

- Working age population ⇒ the population projections produced by Eurostat (Europop) are utilized (updated on an annual basis);
- Trend labour force participation rate and hours worked per person ⇒ in the long-term both series settle at a fixed level (i.e. trend participation rate and hours worked per person converge to a zero contribution to potential growth (i.e. constant levels) by $T+10$);
- NAIRU ⇒ NAIRU remains unchanged from $T+10$, without particularly specifying a level to which it converges;
- Capital stock ⇒ assumption utilizes historical pre-crisis growth rates as a long-term anchor[25];

[24]Current example of the long-term forecasting exercise is of purely technical nature and should not be considered as an official long-term potential output estimate of the Bank of Slovenia.

[25]A balanced growth path (BGP) assumption (i.e. in the long run (i.e. $T+10$), the capital stock grows at the same rate as potential output) turns out problematic for Slovenia, since the country's investment was significantly hit during the crisis.

- Trend TFP ⇒ we assume gradual convergence towards historically observed long-term average (excluding the crisis period). In that sense, we consider historical TFP growth rates as a valuable proxy for long-term TFP growth[26].

Lastly, we present some results from the pseudo real-time exercise[27]. First, we test the expanding window pseudo real-time forecasting ability of our model and compare it to the 4-variable (B)VAR models of order 2 that utilize data on growth rate of GDP, unemployment rate, price inflation and wage inflation[28]. The exercise was conducted using the estimation sample with the pseudo real-time forecasts starting in 2000Q1 in order to strike a balance between the size of estimation and forecasting samples. A desired feature of the extended UCM would be to be able to forecast several macroeconomic variables with at least some degree of accuracy over a monetary policy relevant horizon. The results presented in Table 5 show the average Root Mean Squared Forecast Errors (RMSFEs) for 1- to 12-quarters ahead horizon.

[26] Alternatively, EC (Ageing report) long-term projections may be used.

[27] The expanding window pseudo real-time forecasting exercise is based on a fixed model parametrisation obtained by utilizing full estimation sample (from 1996Q1 to 2018Q4). In other words, the models utilized in the exercise are not re-estimated at each point in time.

[28] In the case of BVAR model, Litterman's prior dummy observation (Litterman, 1979, 1980) was used by utilizing standard hyper-parameter values $\rho = 1$ (random-walk priors), $\mu = \sqrt{N}$ (weight on dummy observations) and $\lambda = 0$ (all lags are weighted equally).

Table 5: Average RMSFE for 1- to 12-quarters ahead horizon forecasts

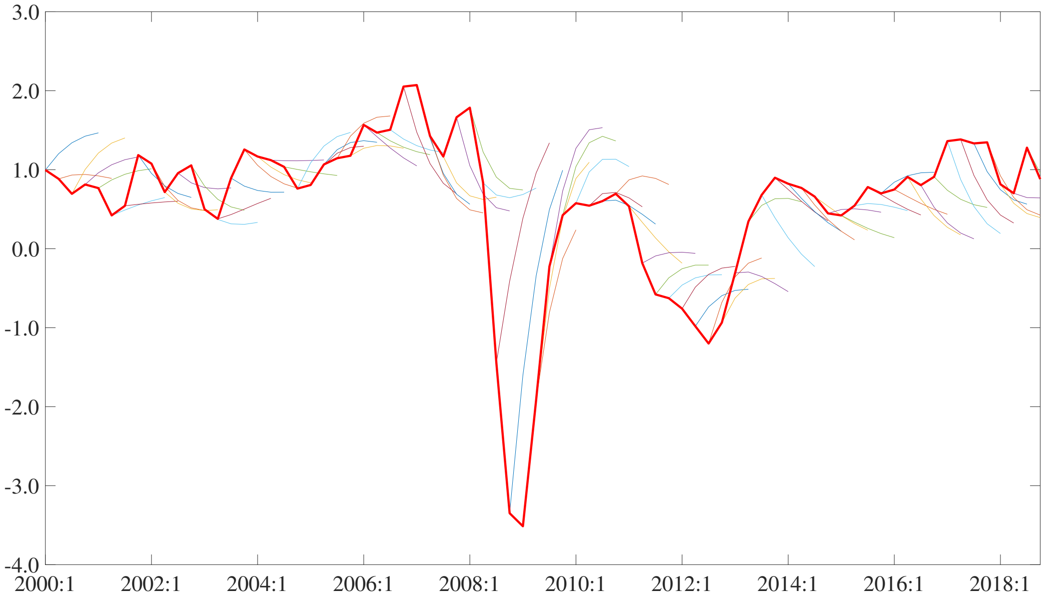
Quart. ahead	GDP growth (q-o-q)	Core infl. (q-o-q)	Wage infl. (q-o-q)	Unemp. rate (q-o-q)	Core infl. (y-o-y)
			Extended UCM		
+1Q	**0.367**	**0.268**	**0.262**	**0.279**	**0.268**
+2Q	**0.496**	**0.312**	**0.354**	0.402	**0.412**
+3Q	**0.583**	**0.342**	**0.408**	0.509	**0.562**
+4Q	**0.644**	**0.363**	**0.441**	0.609	**0.722**
+5Q	**0.688**	**0.381**	**0.465**	0.698	**0.859**
+6Q	**0.728**	**0.396**	**0.487**	0.779	**0.967**
+7Q	**0.764**	**0.408**	**0.506**	0.854	**1.054**
+8Q	**0.804**	**0.422**	**0.519**	0.921	**1.135**
+9Q	**0.839**	0.434	**0.531**	0.990	**1.208**
+10Q	**0.869**	0.447	**0.540**	1.057	**1.276**
+11Q	**0.902**	0.459	0.549	1.120	**1.347**
+12Q	0.942	0.471	0.559	1.179	1.476
			4-variable VAR		
+1Q	0.541	0.316	0.411	0.297	0.503
+2Q	0.620	0.364	0.476	**0.386**	0.686
+3Q	0.686	0.396	0.506	**0.473**	0.888
+4Q	0.743	0.415	0.518	**0.550**	1.042
+5Q	0.789	0.428	0.523	**0.618**	1.153
+6Q	0.824	0.435	0.534	**0.679**	1.231
+7Q	0.850	0.445	0.546	**0.730**	1.299
+8Q	0.870	0.454	0.555	**0.779**	1.355
+9Q	0.889	0.459	0.564	**0.828**	1.397
+10Q	0.907	0.463	0.569	**0.874**	1.430
+11Q	0.924	0.465	0.574	**0.920**	1.454
+12Q	**0.940**	0.467	0.580	**0.966**	1.476
			4-variable BVAR		
+1Q	0.545	0.284	0.426	0.361	0.468
+2Q	0.622	0.320	0.490	0.448	0.594
+3Q	0.691	0.352	0.524	0.534	0.751
+4Q	0.756	0.370	0.535	0.613	0.869
+5Q	0.811	0.387	0.538	0.688	0.973
+6Q	0.853	0.400	0.544	0.758	1.059
+7Q	0.885	0.413	0.547	0.822	1.137
+8Q	0.910	0.424	0.546	0.883	1.208
+9Q	0.932	**0.432**	0.546	0.942	1.267
+10Q	0.950	**0.441**	0.543	0.998	1.323
+11Q	0.967	**0.447**	**0.540**	1.052	1.368
+12Q	0.983	**0.452**	**0.540**	1.103	**1.403**

Note: *Numbers in bold indicate the lowest value when comparing all models.*
Source: *Own calculations.*

RMSFE results suggest that the extended UCM performs reasonably well in the short to medium-term horizon. Interestingly, the model performs consistently well over the 2-year horizon, which is also relevant from a monetary policy perspective (Markov, 2015 and Constâncio, 2018). Regarding the annual inflation forecasting, all models include inflation in quarterly growth rates, however the ability to forecast quarterly rates does not necessarily give much information about annual inflation. For that reason we also provide results for the relevant annual inflation forecasts. They suggest that the extended UCM has some forecasting power in the pseudo real-time experiment over the 2.5-year horizon.

Figure 7: 4-quarters ahead pseudo real-time forecasts

(a) Extended UCM GDP growth q-o-q forecast (q-o-q growth in %)



(b) 4-variable VAR GDP growth q-o-q forecast (q-o-q growth in %)

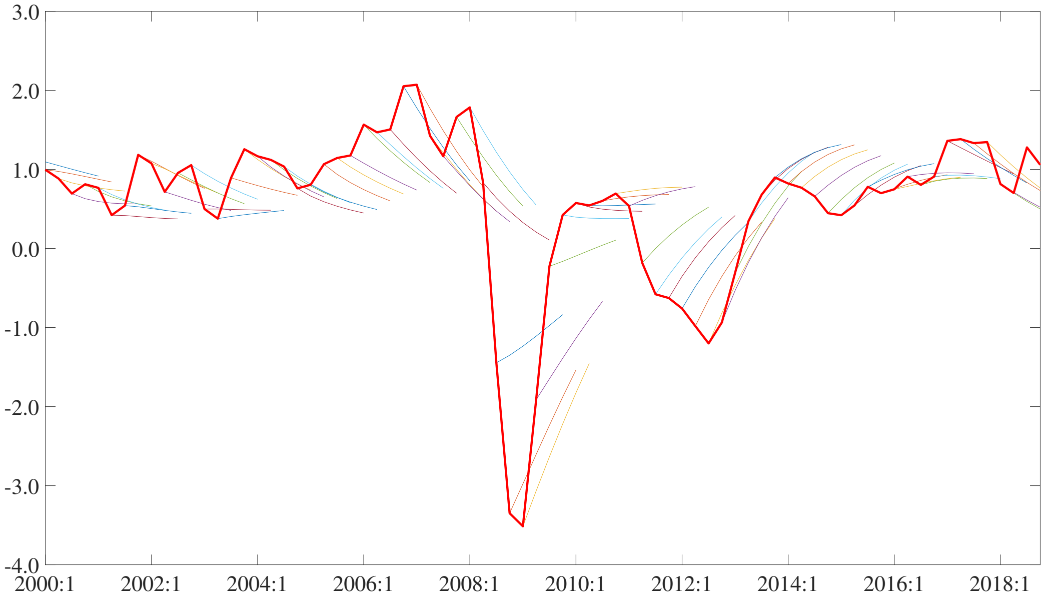
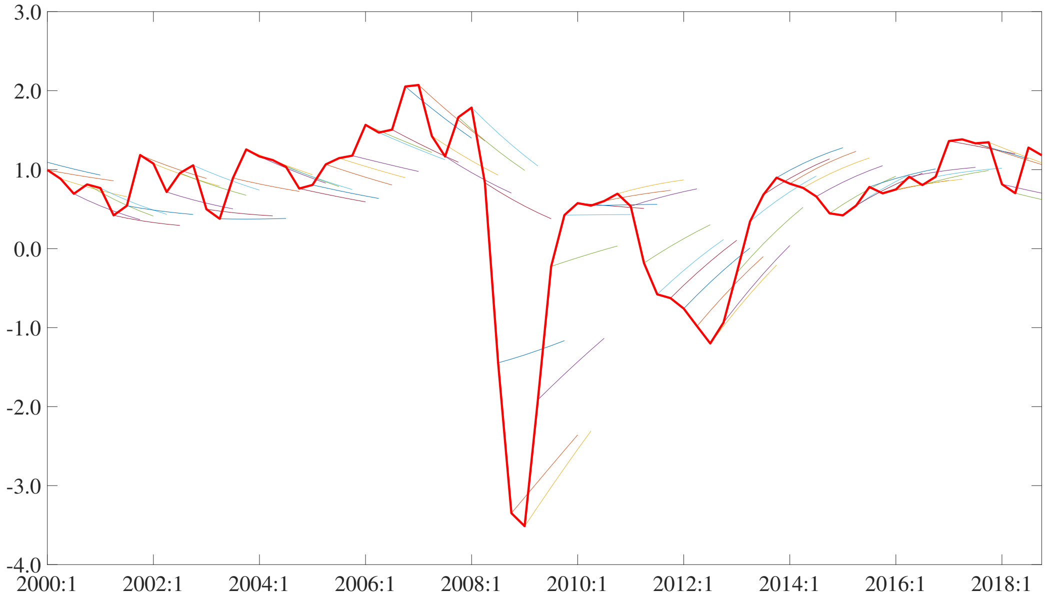


Figure 7: 4-quarters ahead pseudo real-time forecasts (contd.)

(c) 4-variable BVAR GDP growth q-o-q forecast (q-o-q growth in %)



(d) Extended UCM core inflation q-o-q forecast (annualised q-o-q growth in %)

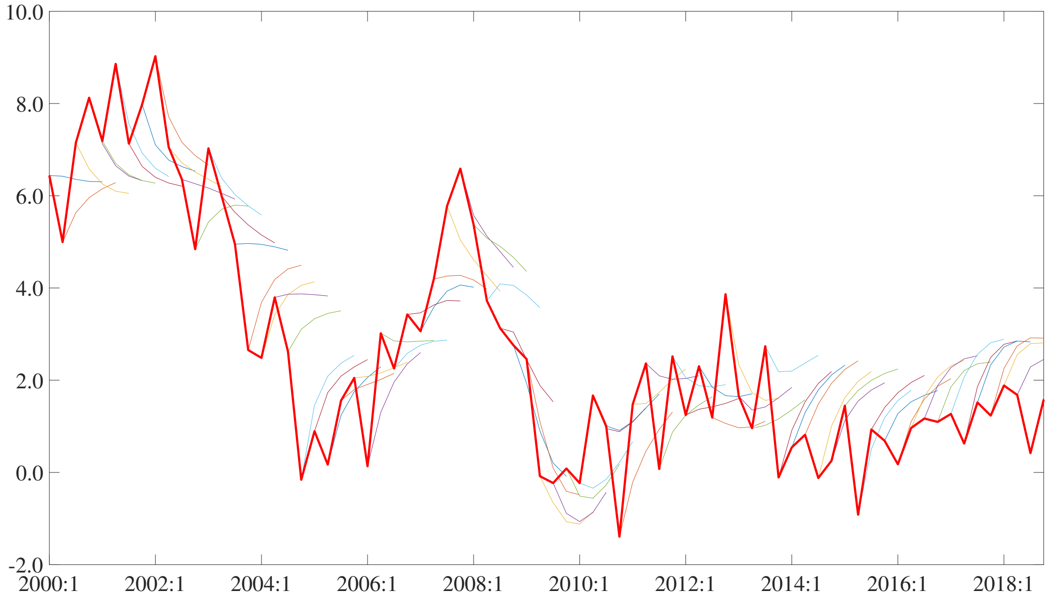
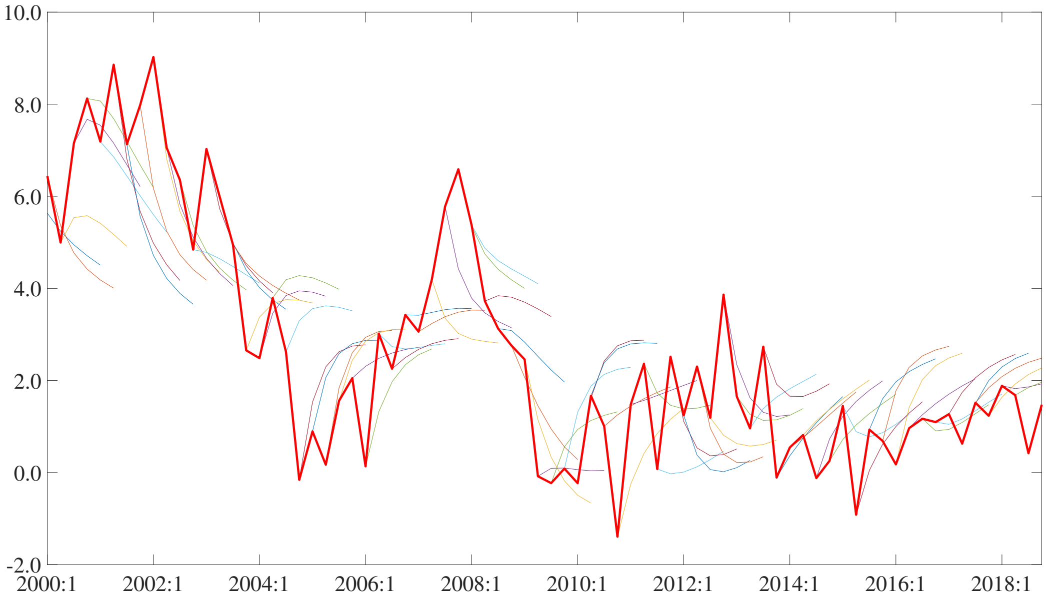
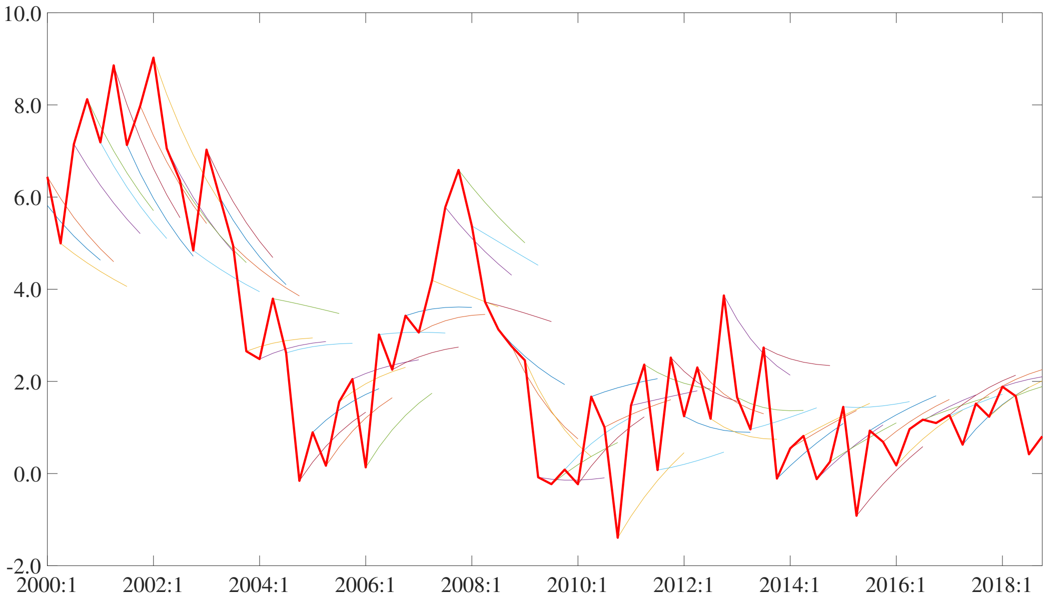


Figure 7: 4-quarters ahead pseudo real-time forecasts (contd.)

(e) 4-variable VAR core inflation q-o-q forecast (annualised q-o-q growth in %)



(f) 4-variable BVAR core inflation q-o-q forecast (annualised q-o-q growth in %)

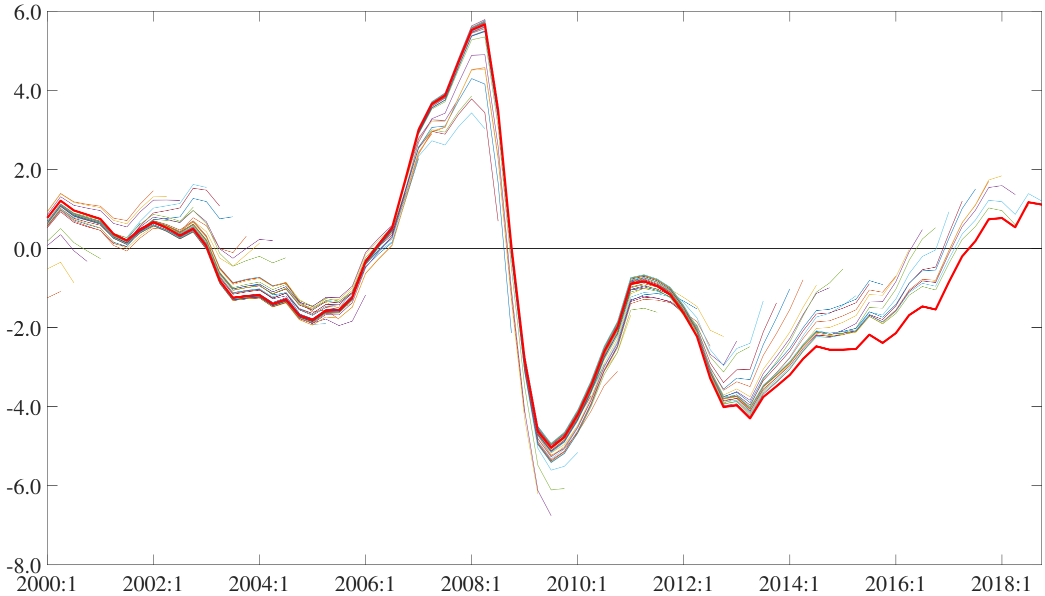


***Note**: Realization (red line) might deviate from the official seasonally adjusted series due to own seasonal adjustment method.*
***Source**: Own calculations.*

The difference in forecasting performance between the selected models is evident also from Figure 7 which shows the 4-quarters ahead pseudo out-of-sample forecasts at different points in time. The extended UCM model seems to perform somewhat better at forecasting GDP growth and price inflation. In addition, when taking into account also other variables[29], there exists some evidence for the forecasting superiority of the extended UCM compared to the conventional 4-variable (B)VAR: the extended UCM therefore seems to contain some policy-relevant information regarding main macroeconomic variables in Slovenia, at least given information that we have now on the importance of extended state-space system in estimating potential output and output gap during the recent financial crisis. In that way, the model could prove valuable in monetary policy related exercises.

Figure 8: Pseudo real-time output gap revisions

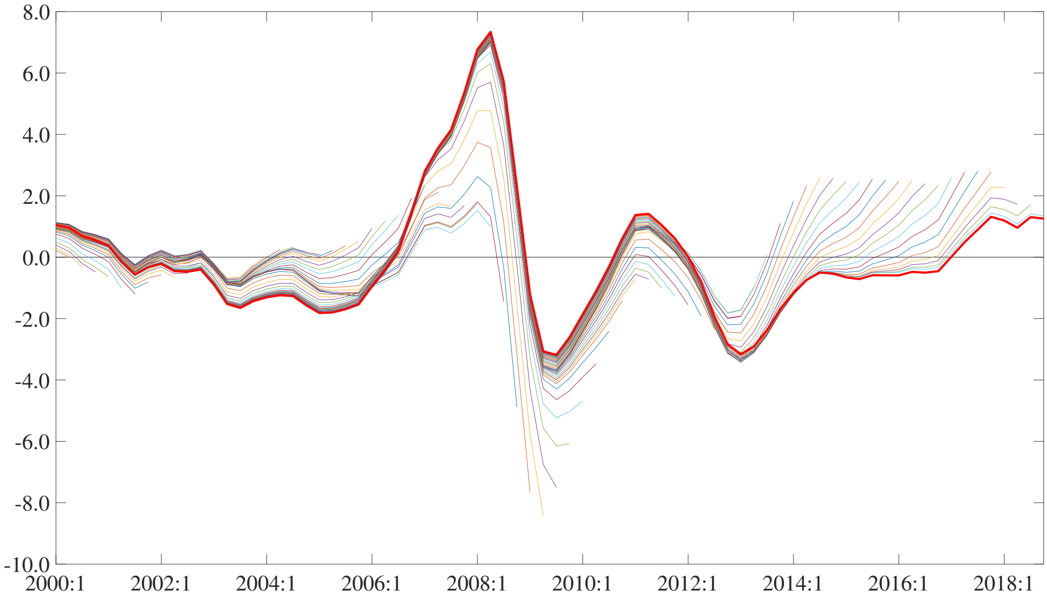
(a) Extended UCM output gap revisions (in % of potential output)



[29]The main part of the text represents only 4-quarters ahead pseudo out-of-sample GDP growth and inflation q-o-q forecasts. Other variables are presented in Figure C.1 in the Appendix C.

Figure 8: Pseudo real-time output gap revisions (contd.)

(b) H-P filter output gap revisions (in % of potential output)



Source: *Own calculations.*

In addition, some forecasting problems may be apparent at the end of sample, also due to the unreliability of the "end-point" estimates of the trend output (Orphanides & van Norden, 2002; Melolinna & Tóth, 2016; Morgan et al., 2019 and Tóth, 2019). Figure 8 therefore compares the pseudo real-time estimates of the output gap between extended UCM (multivariate filter) and H-P filter (univariate filter)[30]. The results suggest that the univariate filters tend to suffer more from the "end-point" problem. As an example, the size of the of the pre-crisis output gap estimates using H-P filter changed substantially as new data became available, which makes decomposition approaches relying on standard univariate filters unsuitable for (pseudo) real-time policy analysis. We have to keep in mind that the "end-point" problem does not necessarily disappear when using more complex methods, however extended UCM seems to provide some improvement regarding pseudo real-time performance simply by exploiting the information content of variables which tend to co-move with the dynamics observed in the output.

[30]H-P filter utilizes traditional smoothing parameter value $\lambda = 1600$ without any additional pure technical out of sample forecasts (e.g. naïve forecasts).

5 Conclusion

The current paper analyses the dynamics of potential output and output gap in Slovenia. For the sake of the research, we develop a semi-structural extended UCM, whose methodology draws on the previous work in applying multivariate filtering techniques. In addition, we extend the traditional multivariate state-space system by utilizing the production function methodology for the estimation of trend output, where we closely follow work done in the WGF Working Group Task Force on Potential Output (Morgan et al., 2019 and Tóth, 2019). Despite its parsimonious structure, extended UCM is able to track the narrative on macroeconomic cycles and trends of the Slovenian economy relatively well, even in the presence of elevated volatility in the crisis and the post-crisis period (also when compared to the estimates of other institutions). The main results of the study show some evidence that embedding important structural relationship between inflation, unemployment and the output gap which are able to mimic developments in the business cycle, tends to produce estimates that are intuitive and consistent with a basic economic theory. In particular, the utilized model is able to identify the pre-crisis build-up and also to successfully pin down the dynamics of Slovenian potential output and output gap in the following years. In that way, the results highlight the importance of the UCM framework for analysing cyclical position of the economy.

The applied production function methodology for estimating potential output also allows us to calculate contributions of different unobservable drivers (trend components of TFP, capital and labour) to the overall potential output growth. Results suggest that over the entire horizon, TFP on average represents the main source of potential output growth. Regarding the crisis period, both TFP and capital were heavily affected as recession influenced long-term technological capacities and caused a collapse of investment activity. In a similar vein, the labour component was marked by a large increase in the unemployment rate and severe cut-down of workforce in some sectors, which resulted in negative contributions of all subcomponents to the overall labour. Post-crisis dynamics reflect improvement in TFP contribution and restoration of production capacities of Slovenian firms, while on the other hand, improvement of the labour component was (and still is) partially dragged down by negative contribution of the working age population caused by mounting problems with population ageing.

In addition to the decomposition of potential output to unobservables, we also analysed the main unobservable gap variables included in the state-space system that drive the smoother output gap estimate. As expected, the economic fluctuations in pre-crisis and crisis years are mainly driven by the developments in the TFP gap, which initially generates a relatively large and persistent positive output gap and subsequently most significantly contributes to the marked slowdown of economic activity during the crisis years. Regarding the second recession wave, a somewhat different reasoning can be used as the economic slowdown is in this case more significantly influenced by unfavourable developments on the labour market which is further reflected in highly persistent negative contribution of unemployment gap. Most recent figures reflect gradual post-crisis recovery in both aforementioned subcomponents.

Regarding the long-term developments, we discuss the results obtained using a set of purely technical long-term assumptions, more specifically we assume explicit future paths for developments in working age population, trend labour force participation rate and hours worked per person, NAIRU, capital stock and trend TFP. Results of the exercise with the extended UCM reflect that considered (purely) technical assumptions produce economically sound long-term narrative.

Lastly, we also present some results of the pseudo real-time forecasting exercise. We first test the expanding window pseudo real-time forecasting ability of our model and compare it to the 4-variable (B)VAR models that utilize data on growth rate of GDP, unemployment rate, price inflation and wage inflation. By using average RMSFEs for the 1- to 12-quarters ahead forecasts, we find that the extended UCM performs reasonably well over the 2-year horizon. In that way, the current model framework seems to contain some policy-relevant information regarding different macroeconomic variables in Slovenia, at least given information that we have now on the importance of extended state-space system in estimating potential output and output gap during the recent financial crisis. In addition, comparison of pseudo real-time output gap revisions shows that multivariate filters tend to suffer less from the "end-point" problem, providing some improvement regarding pseudo real-time performance simply by exploiting the information content of additional variables.

Regarding further research on this topic, the current version of the model could be extended by various explanatory variables as well as alternative measures (or proxies) for different macroeconomic variables. For example model structure can be extended with some financial variables similarly as in Borio et al. (2013, 2014) or Melolinna and Tóth (2016), while when it comes to data, different economic indicators (e.g. various price measures) can be considered. Furthermore, by incorporating an open economy framework we could also take into account global imbalances (similarly as in Alberola et al., 2013), to be able to position our potential output and output gap estimates in a context of international environment.

References

1. Adams, C., & Coe, D. T. (1990). A System Approach to Estimating the Natural Rate of Unemployment and Potential Output for the United States. *IMF Staff Papers, 37*(2), p. 232-293.

2. Alberola, E., Estrada, Á., & Santabárbara, D. (2013). Growth Beyond Imbalances. Sustainable Growth Rates and Output Gap Reassessment. *Banco de España Working Paper 1313*.

3. Alichi, A. (2015). A New Methodology for Estimating the Output Gap in the United States. *IMF Working Paper 15/144*.

4. Alichi, A., Bizimana, O., Laxton, D., Tanyeri, K., Wang, H., Yao, J., & Zhang, F. (2017). Multivariate Filter Estimation of Potential Output for the United States. *IMF Working Paper 17/106*.

5. Andersson, M., Szörfi, B., Tóth, M., & Zorell, N. (2018). Potential Output in the Post-Crisis Period. *ECB Economic Bulletin 7/18*.

6. Apel, M., & Jansson, P. (1999). System Estimates of Potential Output and the NAIRU. *Empirical Economics, 24*(3), p. 373-388.

7. Bank of Slovenia (2017). Economic and Financial Developments (July 2017).

8. Bank of Slovenia (2019a). Financial Stability Report (June 2019).

9. Bank of Slovenia (2019b). Macroeconomic Projections for Slovenia (June 2019).

10. Barro, R., & Sala-i-Martin, X. (2004). *Economic Growth*. Cambridge: The MIT Press.

11. Basistha, A., & Startz, R. (2008). Measuring the NAIRU with Reduced Uncertainty: A Multiple-Indicator Common-Cycle Approach. *The Review of Economics and Statistics, 90*(4), p. 805-811.

12. Baxter, M., & King, R. G. (1999). Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. *Review of Economics and Statistics, 81*(4), p. 575-593.

13. Beffy, P. O., Ollivaud, P., Richardson, P., & Sedillot, F. (2006). New OECD Methods for Supply Side and Medium-Term Assessment: A Capital Services Approach. *OECD Economics Department Working Papers 482*.

14. Beneš, J., Clinton, K., Garcia-Saltos, R., Johnson, M., Laxton, D., Manchev, P., & Matheson, T. (2010). Estimating Potential Output with a Multivariate Filter. *IMF Working Paper 10/285*.

15. Beneš, J., Johnston, M., & Plotnikov, S. (2015). IRIS Macroeconomic Modeling Toolbox. Reference Manual. *IRIS Solutions Team.*

16. Beveridge, S., & Nelson, C. (1981). A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the "Business Cycle". *Journal of Monetary Economics, 7*(2), p. 151-174.

17. Blagrave, P., Garcia-Saltos, R., Laxton, D., & Zhang, F. (2015). A Simple Multivariate Filter for Estimating Potential Output. *IMF Working Paper 15/79.*

18. Borio, C., Disyatat, P., & Juselius, M. (2013). Rethinking Potential Output: Embedding Information about the Financial Cycles. *BIS Working Papers 404.*

19. Borio, C., Disyatat, P., & Juselius, M. (2014). A Parsimonious Approach to Incorporating Economic Information in Measures of Potential Output. *BIS Working Papers 442.*

20. Butler, L. (1996). The Bank of Canada's New Quarterly Projection Model, Part 4. A Semi-Structural Method to Estimate Potential Output. Combining Economic Theory with a Time-Series Filter. *Bank of Canada Technical Report 77.*

21. Cerra, V., Chaman Saxena, S. (2000). Alternative Methods of Estimating Potential Output and Output Gap: An Application to Sweden. *IMF Working Paper 00/59.*

22. Ciccarelli, M., & Osbat, C. (2017). Low Inflation in the Euro Area: Causes and Consequences. *ECB Occasional Paper Series 181.*

23. Cobb, C. W., & Douglas, P. H. (1928). A Theory of Production. *American Economic Review, 18*(1), p. 139-165.

24. Costâncio, V. (2018). Past and Future of the ECB Monetary Policy. In *Speech at Conference on Central Banks in Historical Perspective: What Changed after the Financial Crisis.* Valletta: Central Bank of Malta.

25. Darvas, Z., & Simon, A. (2015). Filling the Gap: Open Economy Considerations for More Reliable Potential Output Estimates. *Bruegel Working Paper 2015/11.*

26. De Masi, P. R. (1997). IMF Estimates of Potential Output: Theory and Practice. *IMF Working Paper 97/177.*

27. Denis, C., Grenouilleau, D., McMorrow, K., & Röger, W. (2006). Calculating Potential Growth Rates and Output Gaps. A Revised Production Function Approach. *European Commission Economic Papers 247.*

28. European Central Bank (2016). A Guide to the Eurosystem/ECB Staff Macroeconomic Projection Exercises.

29. European Commission (2012). European Economy: Macroeconomic imbalances - Slovenia. *Occasional Papers 109*.

30. Engle, R. F. (1979). Estimating Structural Models of Seasonality. In A. Zellner (ed.), *Seasonal Analysis of Economic Time Series*, (p. 281-308). Washington D.C.: Department of Commerce, Bureau of Consensus.

31. Fleischman, C. A., & Roberts, J. M. (2011). From Many Series, One Cycle: Improved Estimates of the Business Cycle from a Multivariate Unobserved Components Model. *FEDS Working Papers 11/46*.

32. Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2014). *Bayesian Data Analysis* (3rd ed.). New York: Chapman and Hall/CRC.

33. Gerdrup, K., Kvinlog, A. B., & Schaanning, E. (2013). Key Indicators for a Countercyclical Capital Buffer in Norway - Trends and Uncertainty. *Norges Bank Staff Memo 13/2013*.

34. Hamilton, J. D. (1994). *Time Series Analysis*. Princeton: Princeton University Press.

35. Harvey, A. C. (1985). Trends and Cycles in Macroeconomic Time Series. *Journal of Business and Economic Statistics, 3*(3), p. 216-227.

36. Harvey, A. C., & Todd, P. H. J. (1983). Forecasting Economic Time Series with Structural and Box-Jenkins Models: A Case Study (with Discussion). *Journal of Business and Economics Statistics, 1*(4), p. 299-315.

37. Havik, K., McMorrow, K., Orlandi, F., Planas, C., Raciborski, R., Röger, W., Rossi, A., Thum-Thysen, A., & Vandermeulen, V. (2014). The Production Function Methodology for Calculating Potential Growth Rates and Output Gaps. *European Commission Economic Papers 535*.

38. Hodrick, R. J., & Prescott, E. C. (1981). Post-War U.S. Business Cycles: An Empirical Investigation. *Carnegie Mellon University Discussion Paper 451*.

39. Hodrick, R. J., & Prescott, E. C. (1997). Post-War U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking, 29*(1), p. 1-16.

40. Jemec, N. (2012). Output Gap in Slovenia. What Can we Learn from Different Methods? *Bank of Slovenia Discussion Papers 04/2012*.

41. Juillard, M., Kameník, O., Kumhof, M., & Laxton, D. (2006). Measures of Potential Output from an Estimated DSGE Model of the United States *CNB Working Paper Series 11/2006.*

42. Kennedy, J, & Eberhart, R. C. (1995). Particle Swarm Optimization. *Proceedings of IEEE International Conference on Neural Networks, 4*, p. 1942-1948.

43. Kim, C. J. & Nelson, C. R. (1999). *State-Space Models with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications.* Cambridge: MIT Press

44. King, R. G., & Rebelo, S. T. (1993). Low Frequency Filtering and Real Business Cycles. *Journal of Economic Dynamics and Control, 17*, p. 207-231.

45. Kuttner, K. N. (1994). Estimating Potential Output as a Latent Variable. *Journal of Business and Economic Statistics, 12*(3), p. 361-368.

46. Laxton, D., & Tetlow, R. (1992). A Simple Multivariate Filter for the Measurement of Potential Output. *Bank of Canada Technical Report 59.*

47. Litterman, R. (1979). Techniques of Forecasting Using Vector Autoregressions. *Federal Reserve of Minneapolis Working Paper 115.*

48. Litterman, R. (1980). A Bayesian Procedure for Forecasting with Vector Autoregression. *Working Papers, Massachusetts Institute of Technology, Department of Economics.*

49. Markov, N. (2015). Actual Versus Perceived Taylor Rules: How Predictable is the European Central Bank? In F. Jawadi & W. Barnet (ed.), *Monetary Policy in the Context of Financial Crisis: New Challenges and Lessons.* Bingley: Emerald Group Publishing.

50. Melolinna, M., & Tóth, M. (2016). Output Gaps, Inflation and Financial Cycles in the United Kingdom. *Bank of England Staff Working Paper 585.*

51. Morgan, J., Szörfi, B., & Tóth, M. (2019). Potential Output in Europe. *manuscript, European Central Bank.*

52. Okun, A. (1962). Potential GNP, its Measurement and Significance. *Cowles Foundation Paper 190.*

53. Orphanides, A., & van Norden, S. (2002). The Unrelaibility of Output Gap Estimates in Real-Time. *Review of Economics and Statistics, 84*(4), p. 569-583.

54. Parker, M. (2017). Global Inflation: The Role of Food, Housing and Energy Prices. *ECB Working Paper Series 2024.*

55. Pelagatti, M. M. (2015). *Time Series Modelling with Unobserved Components.* Boca Raton: Chapman and Hall/CRC.

56. Robert, C. P., & Casella, G. (2004). *Monte Carlo Statistical Methods.* New York: Springer.

57. Rostagno, M., Altavilla, C., Carboni, G., Lemke, W., Motto, R., Guilhem, A. S., & Yiangou, J. (2019). A Tale of Two Decades: the ECB's Monetary Policy at 20. *ECB Working Paper Series 2346.*

58. Sariola, M. (2019). An Unobserved Components Model for Finland - Estimates of Potential Output and NAWRU. *BoF Economics Review 2/2019.*

59. Shi, Y., & Eberhart, R. C. (1998). A Modified Particle Swarm Optimizer. *Proceedings of IEEE International Conference on Evolutionary Computation*, p. 69-73.

60. Tóth, M. (2019). A Multivariate Unobserved Components Model to Estimate Euro Area Potential Output - A Production Function Based Approach. *manuscript, European Central Bank.*

61. Turner, D., Cavalleri, M. C., Guillemette, Y., Kopoin, A., Ollivaud, P., & Rusticelli, E. (2016). An Investigation into Improving the Real-Time Reliability of OECD Output Gap Estimates. *OECD Economics Department Working Papers No. 1294.*

62. Vetlov, I., Hlédik, T., Jonsson, M., Kucsera, H., & Pisani, M. (2011). Potential Output in DSGE Models. *ECB Working Paper Series 1351.*

63. Watson, M. W. (1986). Univariate Detrending Methods with Stochastic Trends. *Journal of Monetary Economics, 18*(1), p. 49-75.

64. Watson, M. W., & Engle, R. F. (1983). Alternative Algorithms for the Estimation of Dynamic Factor, MIMIC and Varying Coefficient Regression Models. *Journal of Econometrics, 23*(3), p. 385-400.

Appendices

A State-space representation of the extended UCM

The state-space system considered in the current paper is of the following form:

$$X_t = BX_{t-1} + Cu_t \tag{A.1}$$

$$Z_t = AX_t + v_t \tag{A.2}$$

where

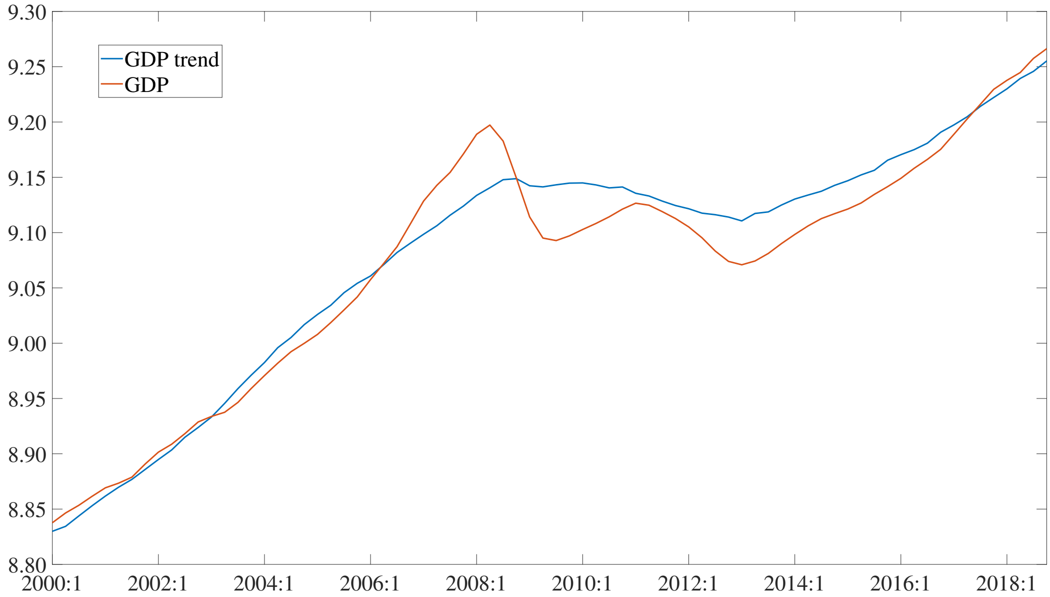
$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim \text{ i.i.d. } N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & H \end{bmatrix}\right) \tag{A.3}$$

Equation A.1 is the state (transition) equation and Equation A.2 is the measurement (observation) equation. X_t is an $(n \times 1)$ vector of unobservable states, which corresponds to the variables, denoted with hats, bars and tildes in Equations 9 to 23, while u_t is an $(m \times 1)$ vector of shocks with $\mathbb{E}\left(u_t\right) = 0$ and $\mathrm{Var}\left(u_t\right) = Q$, which corresponds to the ε_t shocks listed in Table 2. Furthermore, B and C are ($n \times n$ and $n \times m$, respectively) coefficient matrices, which include coefficients from Equations 9 to 23. Regarding the Equation A.1, Z_t is a $(l \times 1)$ vector of observable variables, which corresponds to the left hand side parts of Equations 1 to 8, and A is a $(l \times n)$ selector matrix that combines elements of the state X_t into observable variables. Lastly, v_t is a $(l \times 1)$ vector of measurement errors with $\mathbb{E}\left(v_t\right) = 0$ and $\mathrm{Var}\left(v_t\right) = H$. Since in our case we do not apply any measurement errors to Equations 1 to 8, $v_t = 0$ (constant) and therefore $\mathrm{Var}\left(v_t\right) = 0$.

B Additional figures

Figure B.1: Actual data and trend components

(a) GDP (in logarithms)



(b) Core inflation (annualised q-o-q growth in %)

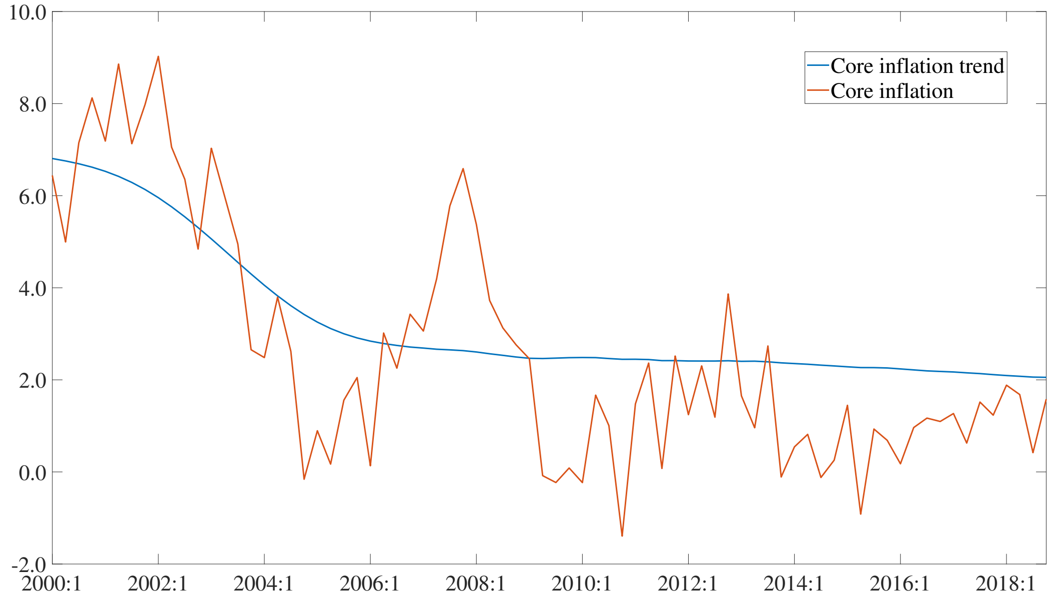
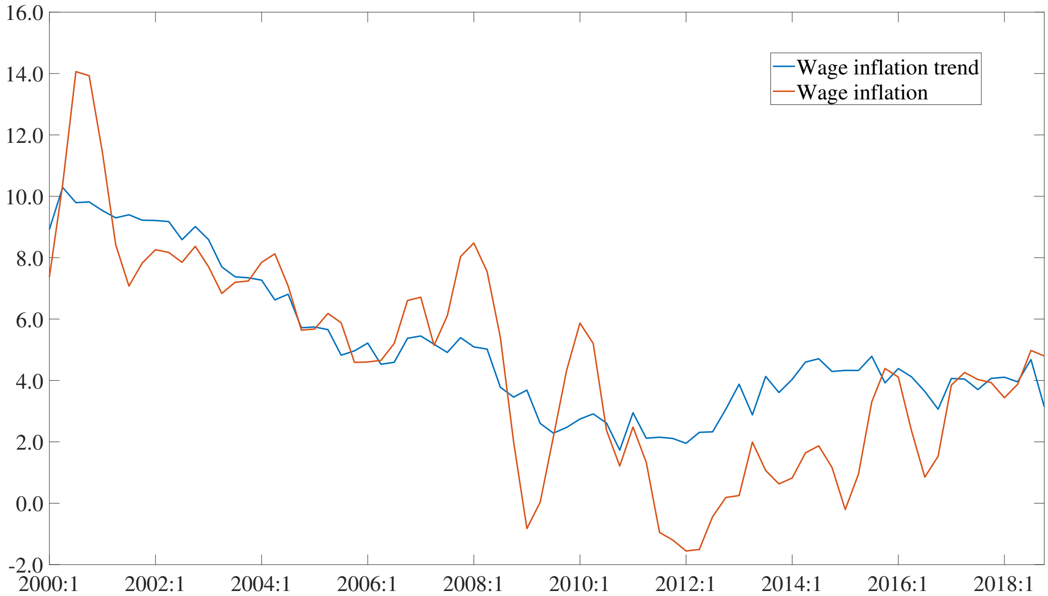


Figure B.1: Actual data and trend components (contd.)

(c) Wage inflation (annualised q-o-q growth in %)



(d) Participation rate (in logarithms)

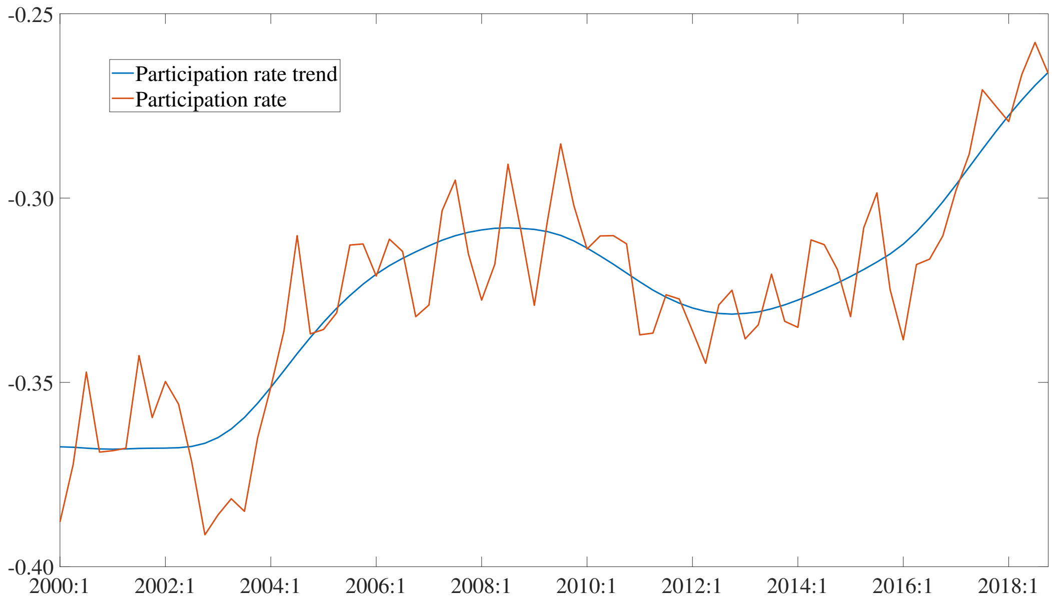
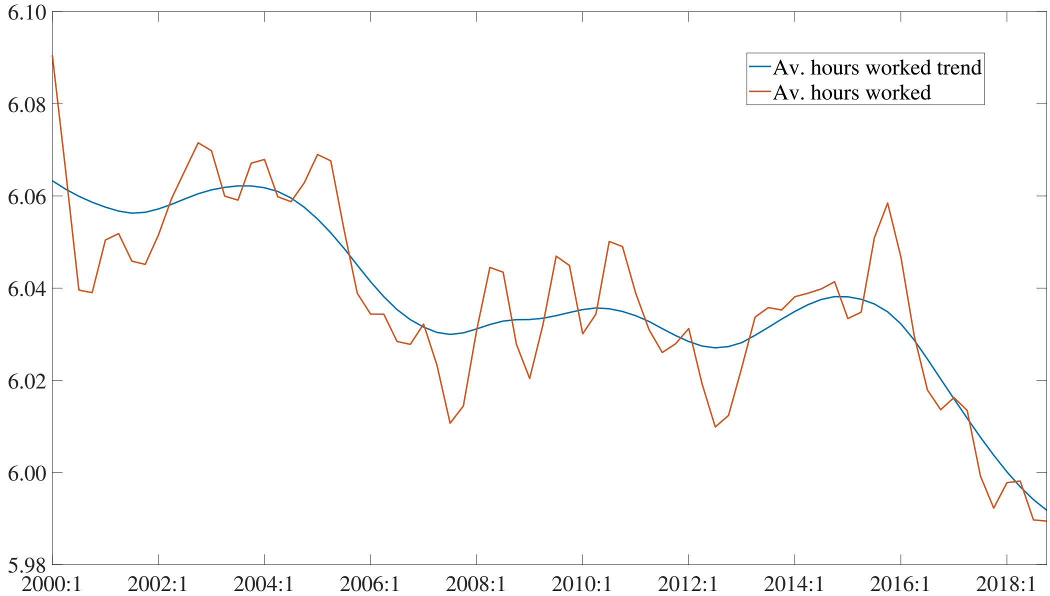


Figure B.1: Actual data and trend components (contd.)

(e) Average hours worked (in logarithms)



(f) Working age population (in logarithms)

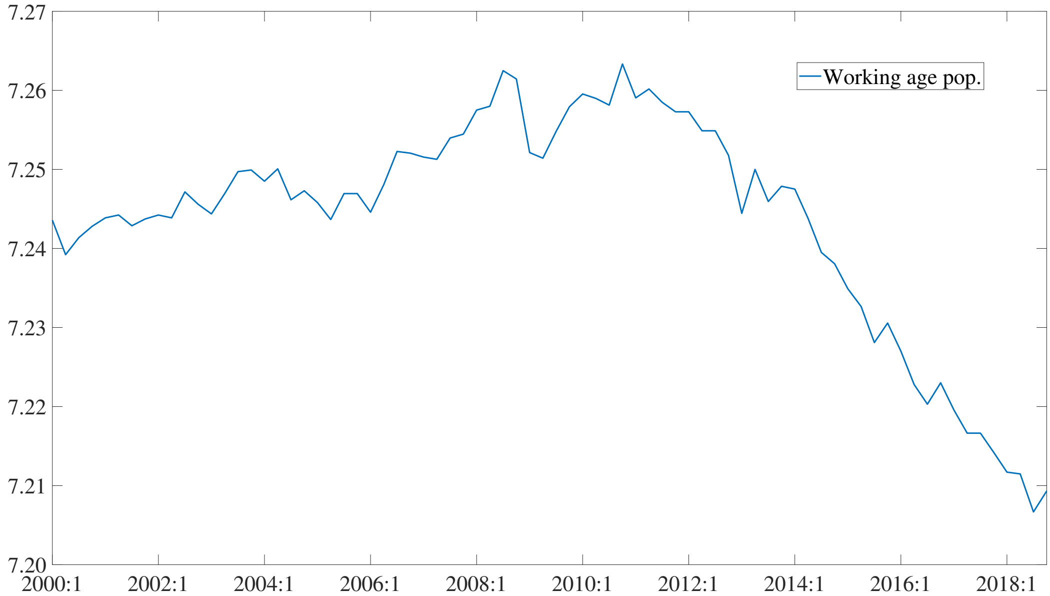
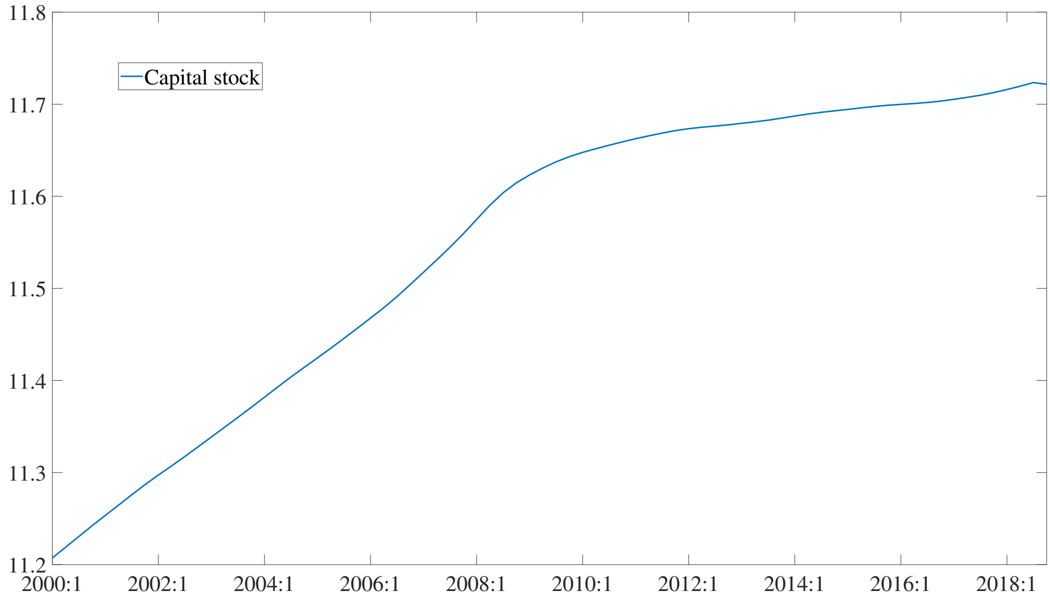
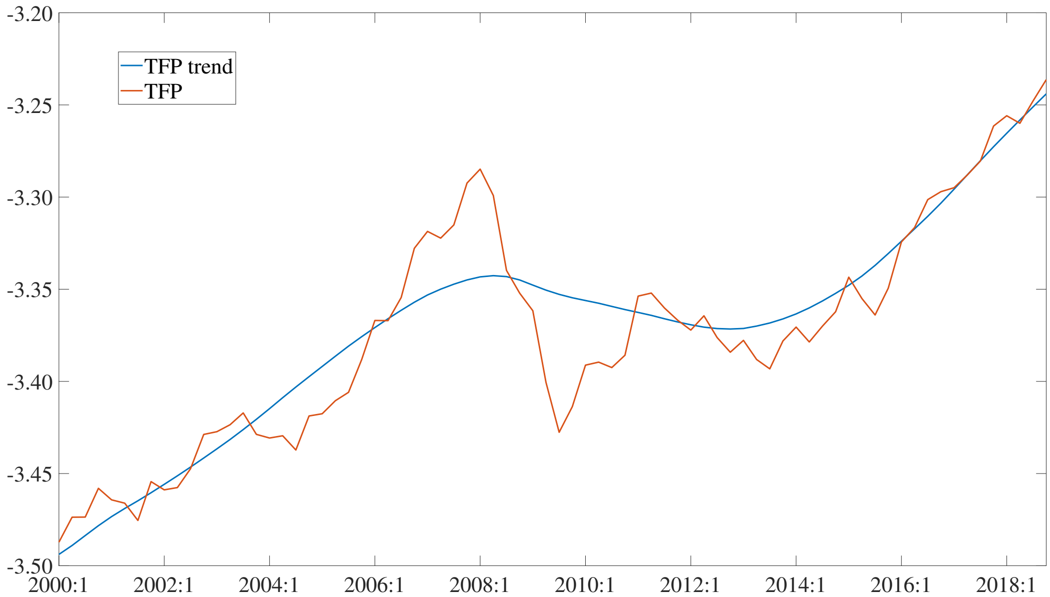


Figure B.1: Actual data and trend components (contd.)

(g) Capital stock (in logarithms)



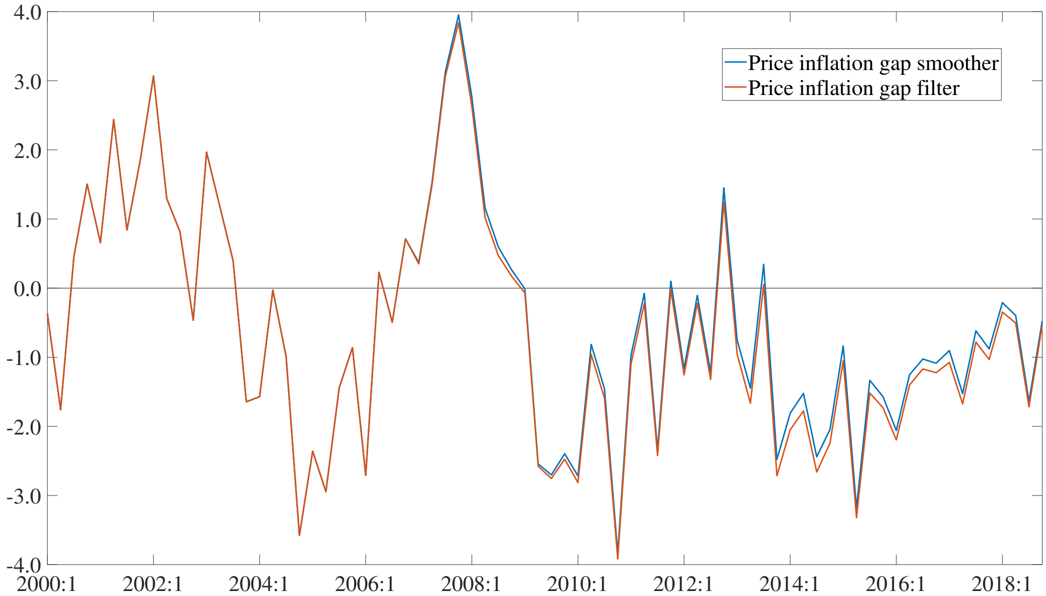
(h) TFP (in logarithms)



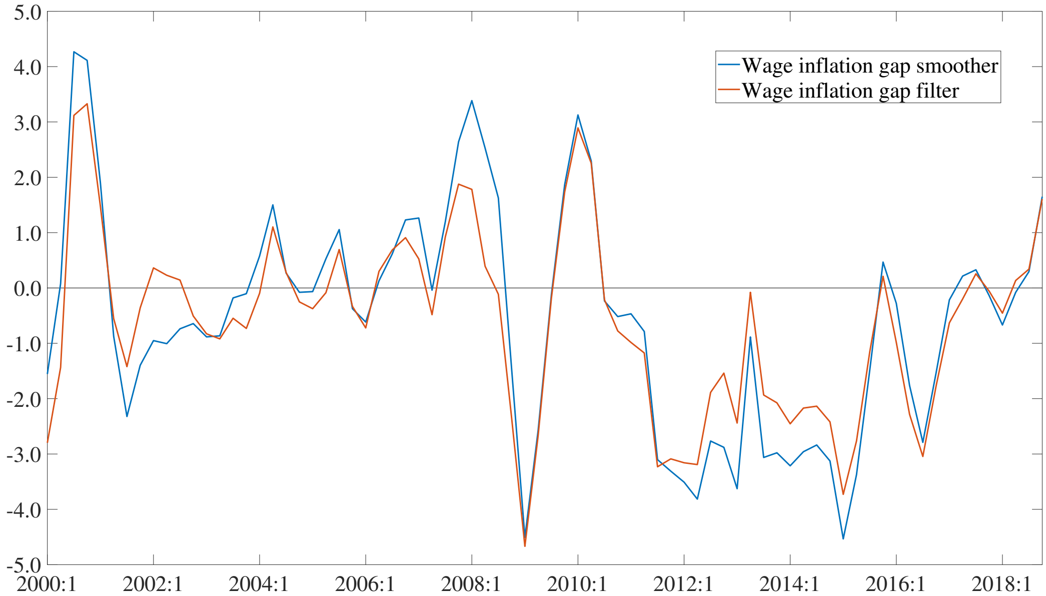
Note: *Realization (orange line) might deviate from the official seasonally adjusted series due to own seasonal adjustment method.*
Source: *SORS, Eurostat, Bank of Slovenia, own calculations.*

Figure B.2: Smoother vs. filter estimates

(a) Price inflation gap (diff. between inflation and trend inflation in %)

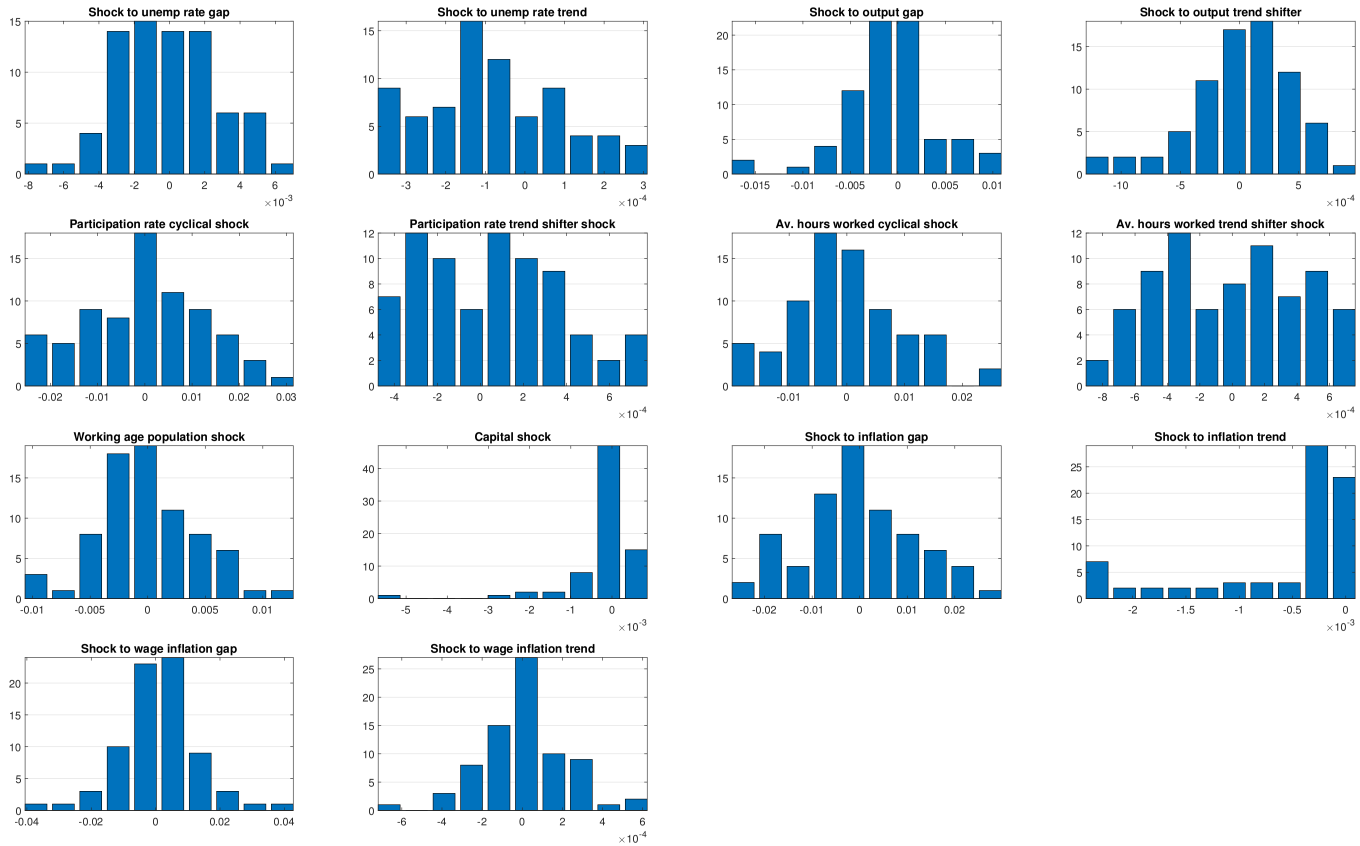


(b) Wage inflation gap (diff. between wage inflation and trend wage inflation in %)



Source: *Own calculations.*

Figure B.3: Histogram of estimated transition shocks



Source: *Own calculations.*

Figure B.4: Prior and posterior distributions

(a) Prior and posterior distributions of parameters

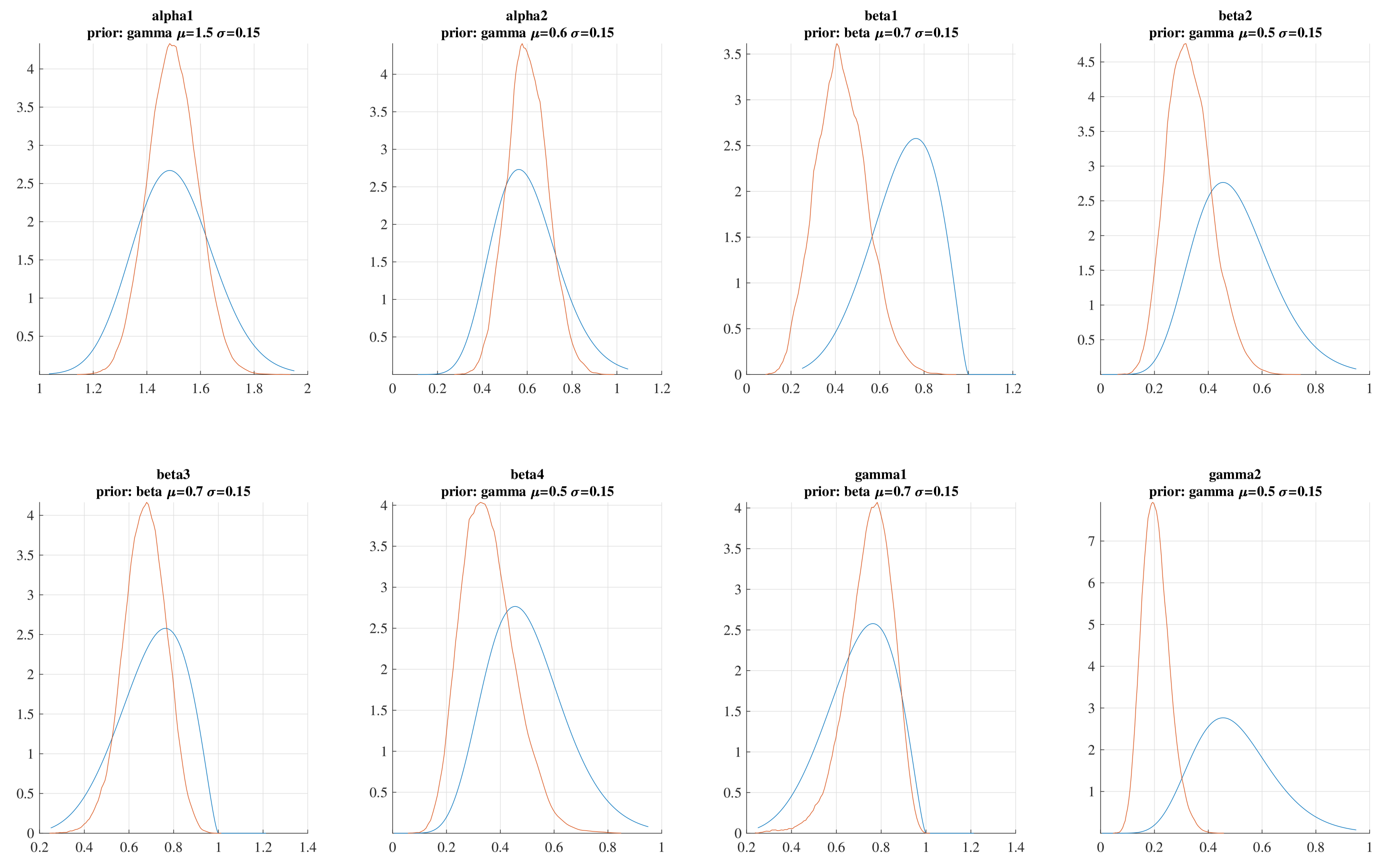


Figure B.4: Prior and posterior distributions (contd.)

(b) Prior and posterior distributions of parameters

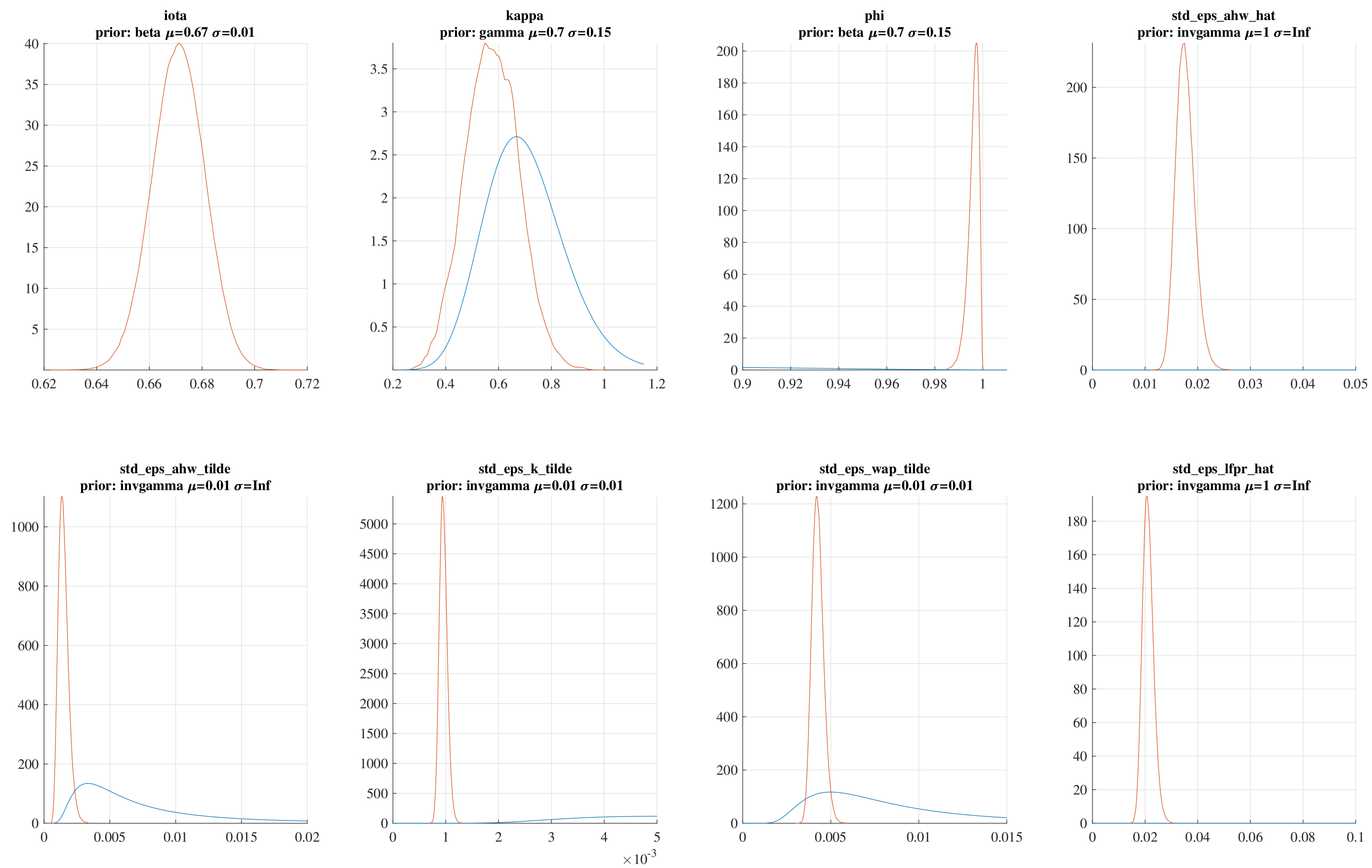


Figure B.4: Prior and posterior distributions (contd.)

(c) Prior and posterior distributions of parameters

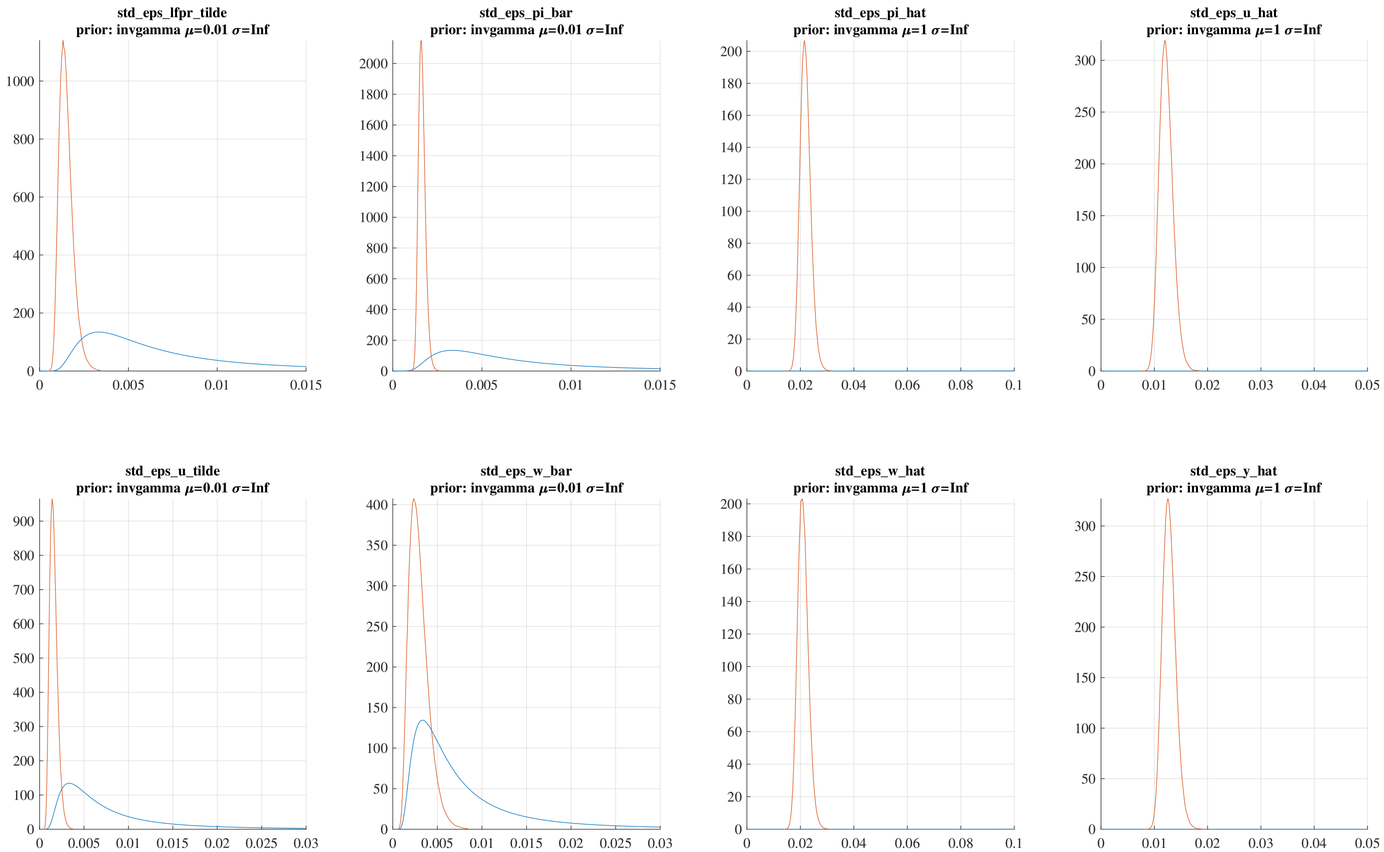
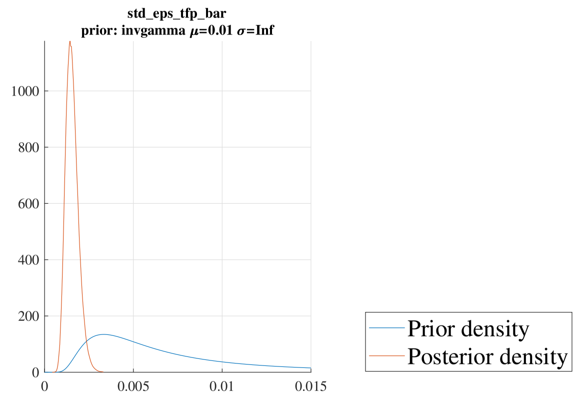


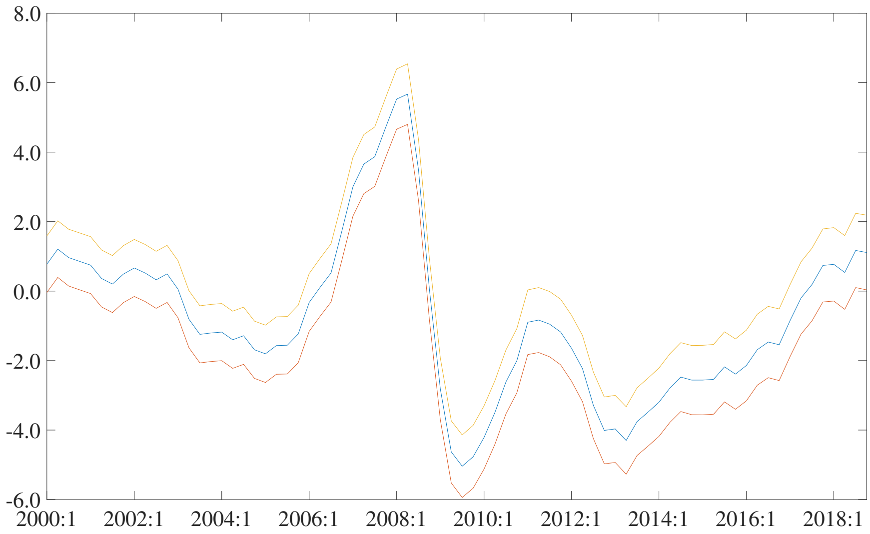
Figure B.4: Prior and posterior distributions (contd.)

(d) Prior and posterior distributions of parameters



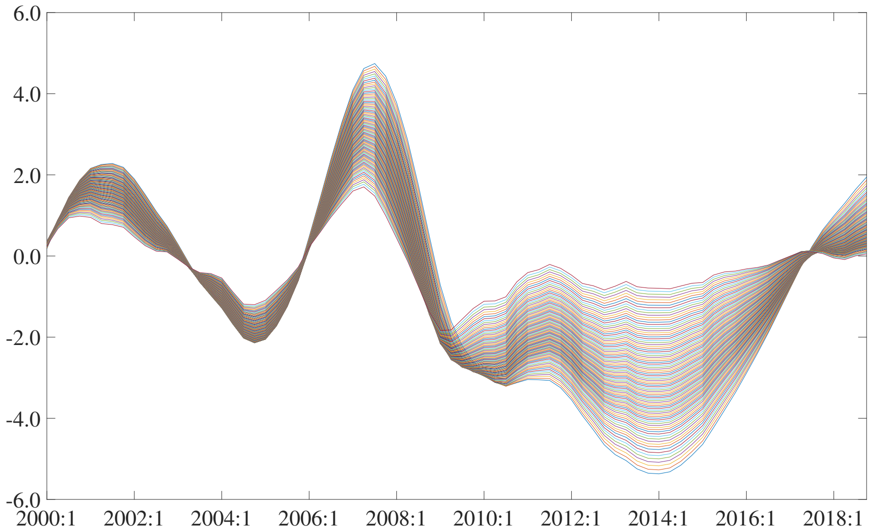
__Source__: Own calculations.

Figure B.5: State uncertainty (one standard deviation) related to output gap estimates (in % of potential output)



Source: *Own calculations.*

Figure B.6: Parameter uncertainty (5-95 percentile) related to output gap estimates (in % of potential output)

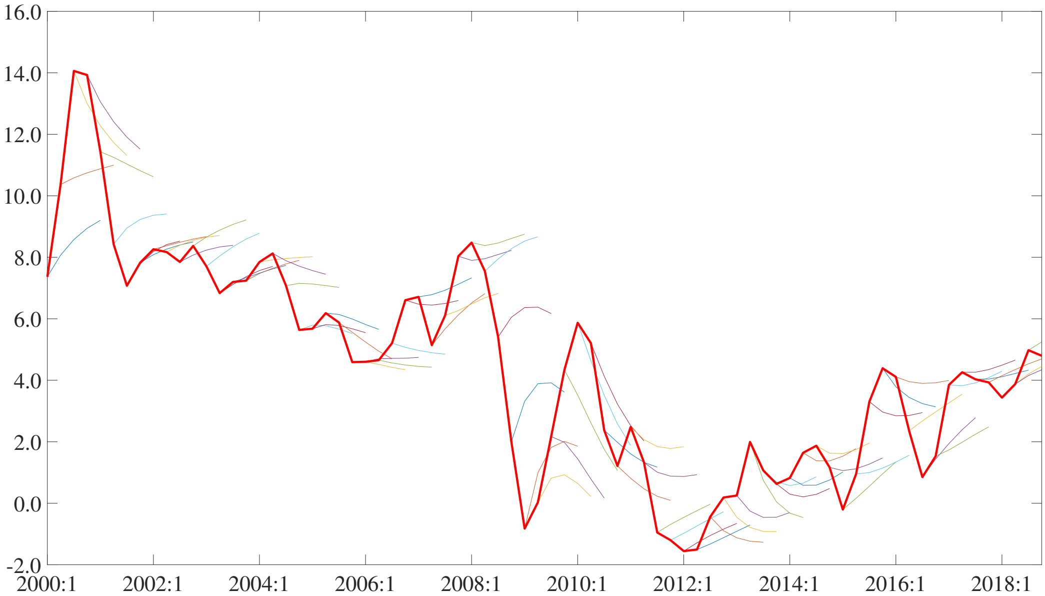


Source: *Own calculations.*

C Additional 4-quarters ahead pseudo real-time forecasts

Figure C.1: 4-quarters ahead pseudo real-time forecasts

(a) Extended UCM wage inflation q-o-q forecast (annualised q-o-q growth in %)



(b) 4-variable VAR wage inflation q-o-q forecast (annualised q-o-q growth in %)

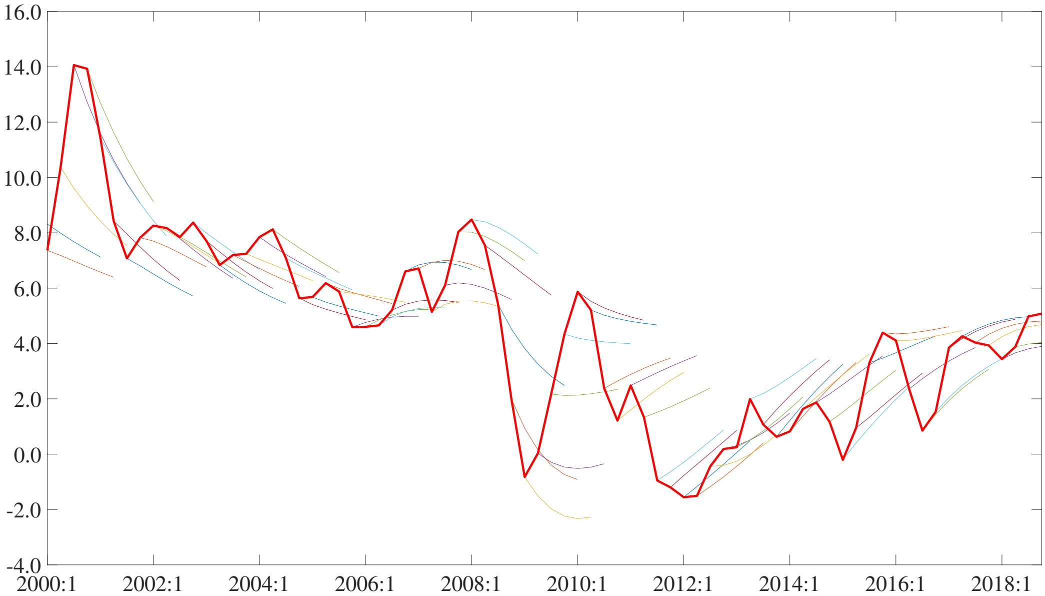
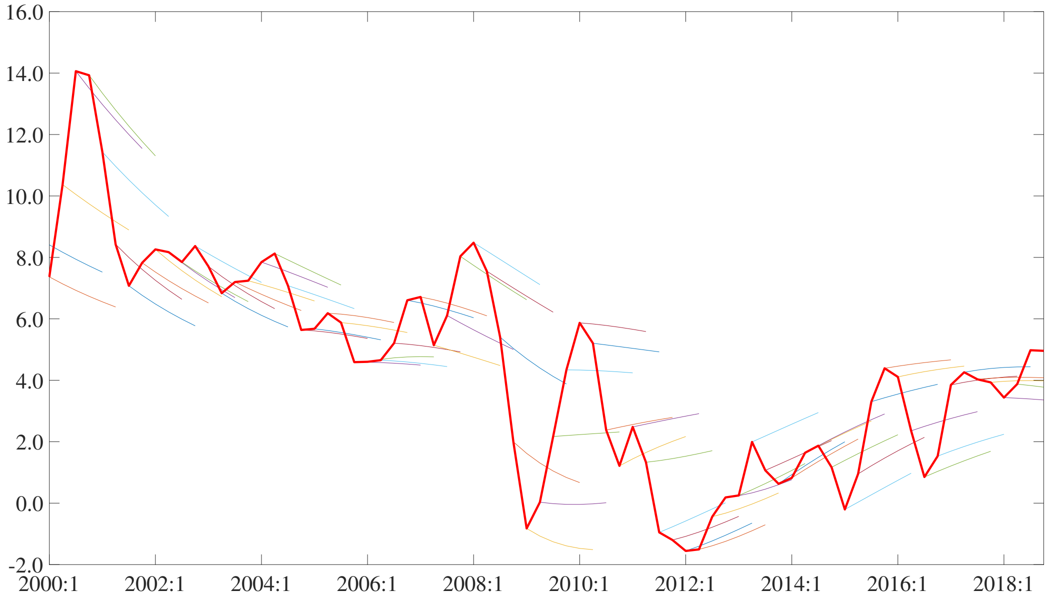


Figure C.1: 4-quarters ahead pseudo real-time forecasts (contd.)

(c) 4-variable BVAR wage inflation q-o-q forecast (annualised q-o-q growth in %)



(d) Extended UCM unemployment rate forecast (in %)

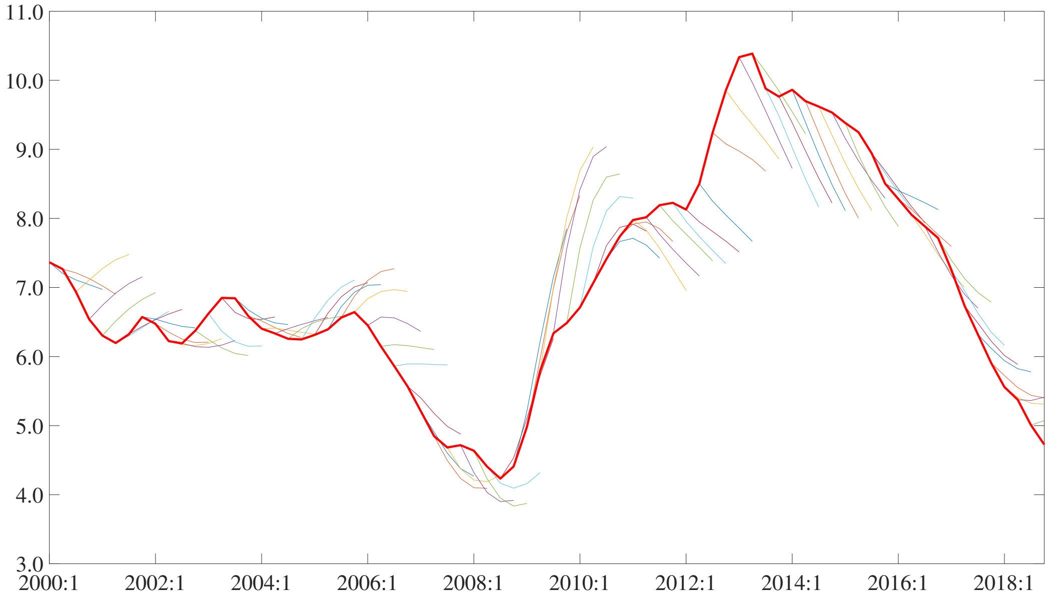
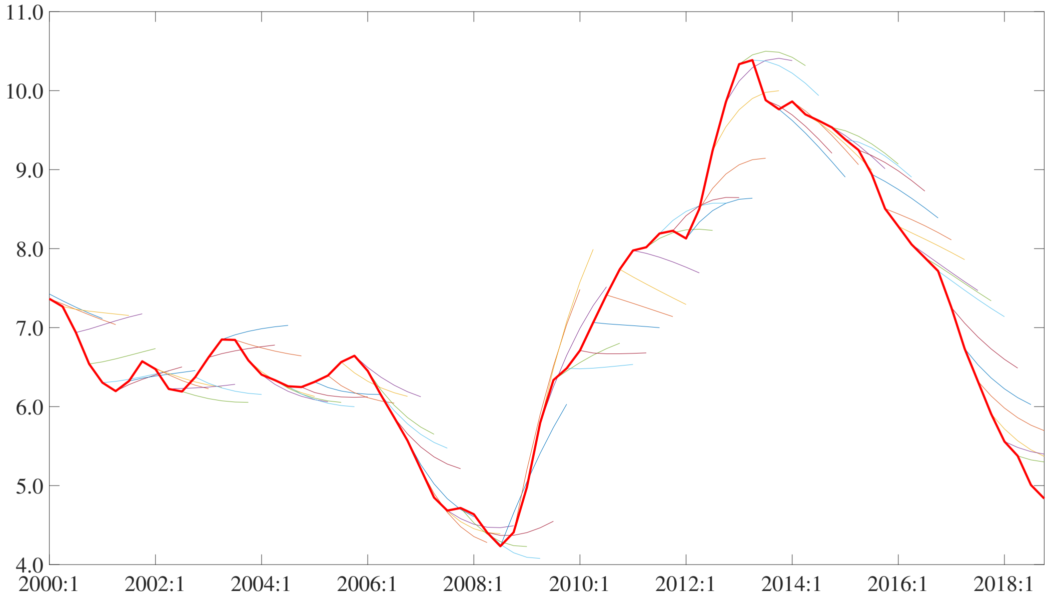


Figure C.1: 4-quarters ahead pseudo real-time forecasts (contd.)

(e) 4-variable VAR unemployment rate forecast (in %)



(f) 4-variable BVAR unemployment rate forecast (in %)

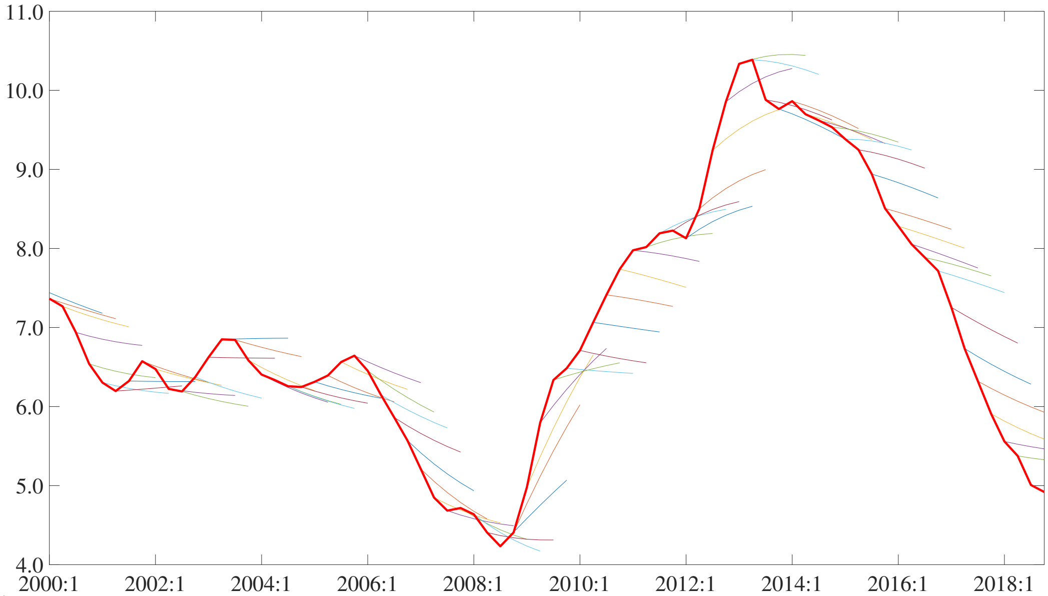
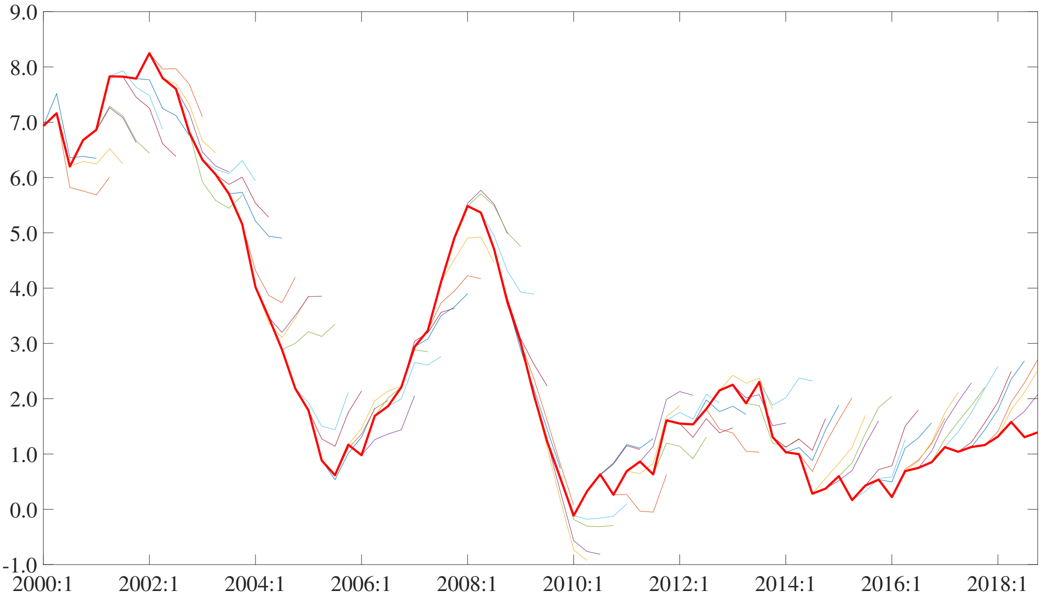


Figure C.1: 4-quarters ahead pseudo real-time forecasts (contd.)

(g) Extended UCM core inflation y-o-y forecast (y-o-y growth in %)



(h) 4-variable VAR core inflation y-o-y forecast (y-o-y growth in %)

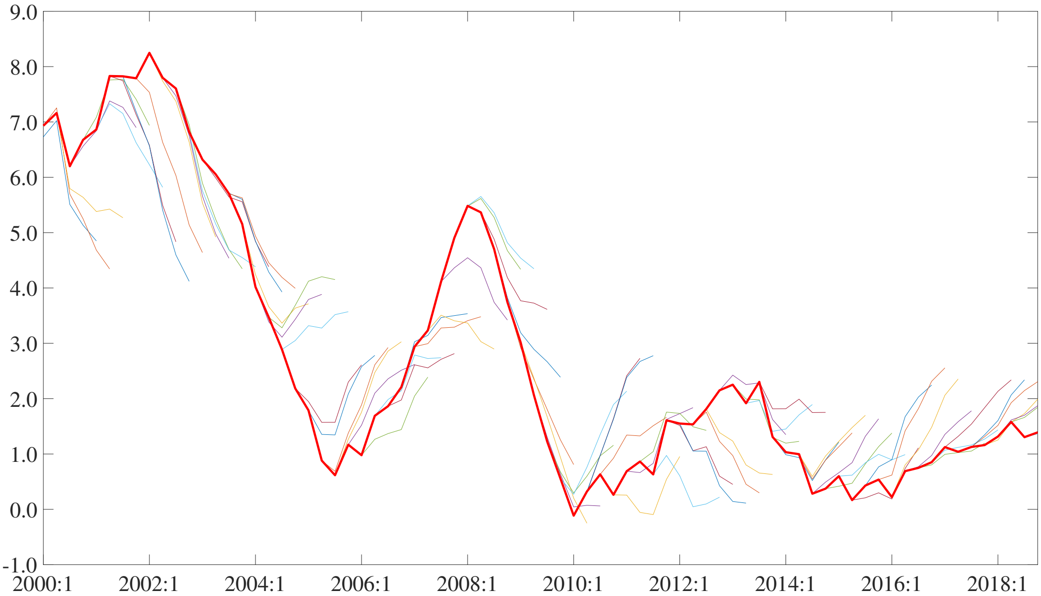
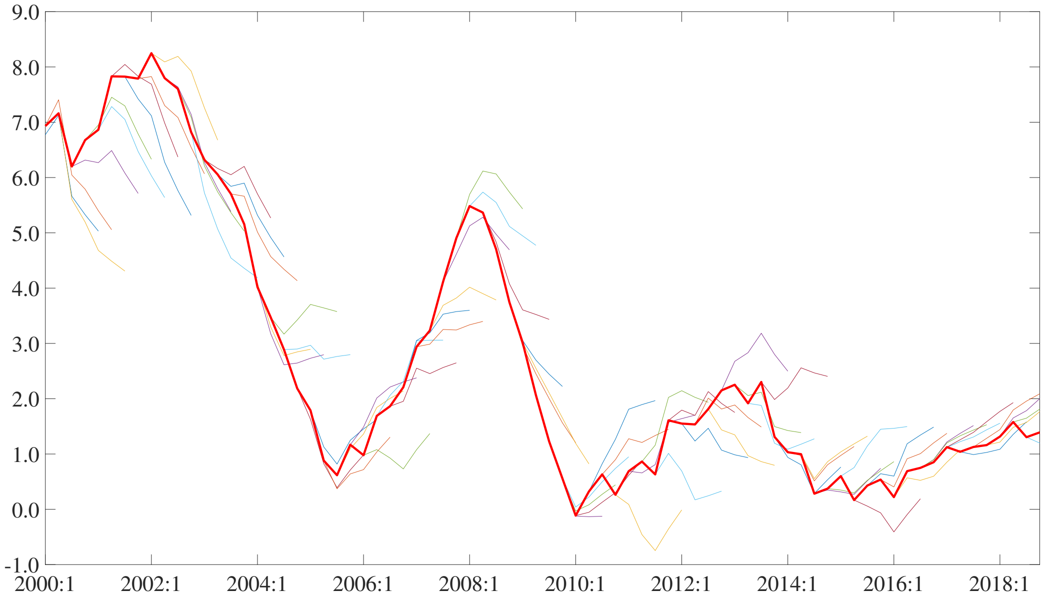


Figure C.1: 4-quarters ahead pseudo real-time forecasts (contd.)

(i) 4-variable BVAR core inflation y-o-y forecast (y-o-y growth in %)



***Note**: Realization (red line) might deviate from the official seasonally adjusted series due to own seasonal adjustment method.*
***Source**: Own calculations.*