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A Note on the Optimal Speed of Transition: Aghion and Blanchard Revisited

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Abstract

This note illustrates, by reconsidering the seminal optimal speed-of-transition model of Aghion and Blanchard (1994), that optimal transition paths, in general, exhibit nonlinearities and discontinuities. Aghion and Blanchard consider only an approximate solution with a constant unemployment rate over the transition process. The exact solution features an increasing unemployment rate with a discontinuity when the state sector is closed down at the optimally chosen endpoint of transition. Economic transition problems bear many similarities to scrap value problems with free terminal time, often encountered in resource economics. In relation to the transition to a green economy, the discussion in this note therefore casts doubt on the optimality of a green transition discussed in, e. g., the European Union in terms of politically specified rather than optimally designed milestones for emissions reductions, i. e., by -55% compared to 1990 levels until 2030 and net zero until 2050.

JEL Classification: C61, E61, P20

Keywords: Dynamic Optimization, End-of-Transition, Optimal Unemployment Rate, Transition

Povzetek

Ta prispevek s ponovnim pregledom temeljnega modela optimalne hitrosti prehoda Aghiona in Blancharda (1994) ponazarja, da imajo optimalne poti prehoda na splošno nelinearnosti in diskontinuitete. Aghion in Blanchard obravnavata le približno rešitev s konstantno stopnjo brezposelnosti v procesu prehoda. Natančna rešitev ima naraščajočo stopnjo brezposelnosti z diskontinuiteto, ko se državni sektor zapre na optimalno izbrani končni točki prehoda. Problemi ekonomskega prehoda so v marsičem podobni problemom vrednosti ostankov s prostim končnim časom, ki se pogosto pojavljajo v ekonomiki virov. V zvezi s prehodom na zeleno gospodarstvo se v tem primeru pojavijo vprašanja o optimalnosti zelenega prehoda, o katerem se razpravlja npr. v Evropski uniji, v smislu politično določenih, namesto optimalno zasnovanih mejnikov za zmanjšanje emisij, tj. za 55

1 Introduction

The history of economic development has repeatedly seen and continues to see fundamental transitions between partly drastically different economic (and political or technical) regimes. In this perspective, the first major economic transition was probably the transition from nomadic societies to agrarian, non-nomadic societies. Important major transitions in more recent centuries include, of course, the (first) industrial revolution and more recently, starting in the early 1990s, the transition to market economies in (most of) the former centrally planned communist countries. This process started with the demise of the Soviet Union and has brought quite rapid (not only) economic change to numerous countries.¹ However, history has not come to an end. Further transition processes are ongoing and can be expected to exert major impacts. These include, e. g., new waves of technological change labelled as “digitalization”, “industry 4.0” or “internet of things” (see, e. g., Brynjolfsson and McAfee, 2011), which can be considered a 21st-century version of an industrial revolution. Another important transition process – very likely the most important transition of our time – is the transition towards an (essentially greenhouse gas) emissions-free economy necessary to limit the detrimental effects of anthropogenic climate change. This so-called *green transition* requires, in particular, fundamental changes in the production (and consumption) of energy, i. e., a replacement of carbon-based energy sources by carbon-neutral energy sources. In this process, energy usage will shift strongly towards electricity to be generated from emissions-free sources. Clearly, the redesign of the global energy infrastructure and system (see, e. g., International Energy Agency, 2023) will have profound impacts on all sectors and potentially also on the composition of the global economy.²

Notwithstanding the heterogeneity of the scopes and impacts of the transition processes mentioned, all these transitions necessitate or imply major reallocations of production factors, in particular also of labor, from *old* sectors to *new* sectors. The importance and magnitude of these processes makes an efficient design imperative. Conceptually, an optimal transition policy is hereby defined in terms of both an optimal speed of transition and an optimal endpoint or, equivalently, an optimal duration of a transition process. We illustrate these

¹For a recent discussion concerning the *partly* ongoing transition processes from centrally planned to market economies, see, e. g., Dabrowski (2023).

²A pivotal contribution describing policy needs for combatting climate change is the report of Sir Nicholas Stern, see Stern (2007) and, for an assessment of the developments since the original publication, Stern (2015). Hassler *et al.* (2016) provide an overview over macroeconomic modelling of climate change and resource scarcity.

two dimensions by deriving in detail *both* the optimal speed as well as the optimal endpoint of a transition process by reconsidering the well-known speed-of-transition model of Aghion and Blanchard (1994) that deals with the transition from a centrally planned towards a market-based economy. More specifically, Aghion and Blanchard (1994, Section 6.4) present a dynamic optimization model to determine the optimal speed of transition, which in their model corresponds to finding the optimal path of the unemployment rate (see also the discussion in Roland, 2000).³ When solving the dynamic optimization problem, Aghion and Blanchard (1994) do not, however, derive the exact solution, but only an “approximate” solution that neglects the behavior of the economy after the state sector has been closed down. Due to the dynamic nature of the economy, however, the post-transition economic performance influences the optimal behavior already during the transition process and thus influences the optimal path also whilst the state sector still employs people.⁴ In this respect, Aghion and Blanchard (1994, Footnote 33, p. 305) state that they are “cheating” by setting certain quantities constant, which they label “turnpike” approximation.

The exact solution differs from the approximate solution in two related ways: First, the optimal unemployment rate is not constant, but increases over time and exhibits a discontinuity when the state sector is closed down at an optimally chosen endpoint of the transition. Second, we find a higher optimal unemployment rate than Aghion and Blanchard, which implies a shorter optimal duration of the transition process, i. e., an earlier endpoint. In the Aghion and Blanchard (1994) model, the inefficiency created by the approximate solution with a constant unemployment rate is that a *too low* unemployment rate reduces the rate of job creation in the new sector, which slows down output growth in this more productive sector and extends the duration of the transition process.⁵

³For a detailed discussion concerning labor market dynamics in transition economies see, e. g., Boeri (2000). Our contribution here is of a conceptual or methodological nature and, thus, several aspects of labor market dynamics that are found to be relevant from a labor economics perspective are neglected, as in the model of Aghion and Blanchard (1994). Also, of course, this type of model has to be interpreted in a stylized fashion with respect to the role of the state in an economy. In the model the state sector is closed down entirely, i. e., the non-trivial role of state sectors also in market economies is, for simplicity and to focus on one aspect, abstracted from. For the same reason, i. e., to focus on one aspect, we also abstract from reform uncertainty and potential reform reversal, issues discussed in Fernandez and Rodrik (1991) or Dewatripont and Roland (1995).

⁴The behavior of the economy at the point in time when the state sector is closed (and thereafter) is neglected also in other speed of transition models: Brixiova and Yousef (2000) assume a constant closure rate of the state sector, which may also lead to different dynamic behavior and welfare losses compared to optimal closure. Burda (1993) also finds a constant optimal unemployment rate, where again the effect of state sector closure is not analyzed in detail. Castanheira and Roland (2000) avoid the problem by assuming that there is no unemployment and that capital can be moved freely from the old to the new sector.

⁵As already mentioned, our analysis is of a merely conceptual and qualitative nature, but a rising unem-

The transition mechanism described here – or in Aghion and Blanchard (1994) – is conceptually closely related to important aspects (that need to be adequately detailed in fully specified models) of the other transition processes mentioned above. This is obvious, e. g., for the mentioned 21st-century industrial revolution, by simply replacing the terminology state and private sector with old and new sector. There are also close links to the green transition, when replacing the labels state and private sector with dirty and clean sector. In the context of the green transition, the government, once it internalizes the negative climate externality of the dirty sector (with a higher private but lower social marginal product than the clean sector), faces the problem of managing an optimal transition to an effectively carbon emissions free economy.⁶

This short paper is organized as follows: In Section 2, we set up and analyze the Aghion and Blanchard (1994) model in detail and Section 3 draws some conclusions.

2 The Aghion-Blanchard Model: Exact Solution for Normative Analysis

We focus on the dynamic optimization problem used for a normative analysis of a transition process in Aghion and Blanchard (1994, Section 6.4) and present only those parts of the model presented in their paper in detail that are of immediate relevance here.

Denote with $E(t)$ the number of people employed in the state sector (with constant marginal productivity x), with $N(t)$ the number of people employed in the emerging private sector (with constant marginal productivity $y > x > 0$) and with $U(t)$ the number of unemployed people at time t . Population is normalized to one, i. e., $E(t) + N(t) + U(t) = 1$, which implies that $U(t)$ is both the number of unemployed people and the unemployment rate. Aghion and Blanchard (1994) develop an efficiency wage-based explanation for costly labor adjustment between the old state sector and the new private sector. In particular, they

ployment rate during an optimal transition process increases the risk of *costly* reform reversals and backlashes. This is an issue that, however, cannot be addressed when considering a central planning solution only.

⁶The analogy to the green transition has to be considered more carefully also in terms of modelling stocks and flows: A key aspect to be included in climate-economy transition models is that they need to take into account the (uncertain) negative impacts of the *accumulated stock* of emissions on all sectors in the economy via a *damage function* of some sort, see, e. g., Hassler *et al.* (2024) for a recent policy-oriented discussion. Furthermore, the different transitions listed are, of course, intertwined, with, e. g., the direction and speed of technical change not independent of climate policies, see, e. g., Hassler *et al.* (2022).

Furthermore, in the context of environmental problems there is a well-known discussion about instrument choice in environmental economics (quantity constraints, taxation,...). Whichever instrument chosen, the result will be a shrinking of the dirty sector that has to be managed by choosing optimal policy paths.

derive the following relationship for the speed of job creation in the new private sector (see their equation (9) on page 298):⁷

$$\dot{N} = f(U) = a \left[\frac{U}{U + ca} \right] \left[y - rc - \left(\frac{b}{1 - U} \right) \right], \quad (1)$$

with a, b, c and r positive constants. Here, a indicates the impact of per-worker profits in the private sector on the speed of private-sector job creation, b are unemployment benefits and c is a constant related to the “wage-premium”, derived using an efficiency-wage setting, that private firms are willing to pay (over and above unemployment benefits as outside option). Furthermore, r is the interest rate and the cost of job creation in the private sector is given by $\frac{1}{2ar}(f(U))^2$.⁸ The government chooses the optimal speed of closure of the inefficient state sector and, thereby, unemployment.

Remark 1 *It may be interesting to discuss the building blocks leading to private-sector job creation as given in (1) in a bit more detail: This relationship is based on the combination of three modelling assumptions with an efficiency-wage consideration. First, Aghion and Blanchard (1994) assume that the rate of private-sector job creation is proportional to profits per worker, i. e., $\dot{N} = a(y - z - w)$ with y being the (constant) average product of labor in the private sector, z taxes per worker, w the private-sector wage and the proportionality constant a , compare (2) in Aghion and Blanchard (1994). Second, private-sector wages w depend upon labor market conditions as follows $w = b + c(r + \frac{\dot{N}}{U})$, with unemployment benefits b , the interest rate r , $\frac{\dot{N}}{U}$ the ratio of new jobs to unemployment and a constant c that scales the wage premium over unemployment benefits that firms are willing to pay, compare (3) in Aghion and Blanchard (1994).⁹ The third element is that unemployment benefits are financed by labor taxes under the condition of a balanced budget, i. e., $Ub = (1 - U)z$, see (6) in Aghion*

⁷To avoid overloaded notation we sometimes skip the time index t . Furthermore, $\dot{N}(t)$ denotes the derivative of $N(t)$ with respect to t , with this notation also used for other variables.

⁸Another restriction on the parameters is $y - b - rc > 0$. In this case, $f(U)$ is positive for values of U larger than zero and smaller than $\frac{y-b-rc}{y-rc}$. Clearly, it cannot be optimal to consider unemployment paths that include values larger than this value with negative rates of job creation in the more productive private sector.

⁹The relationship for private-sector wages is developed in Aghion and Blanchard (1994) in an efficiency-wage framework under the assumptions that all hires are from unemployment and that once employed in the private sector there is no risk of future unemployment. Thus, the value of being unemployed, V_U , is given, see (4) and (5) in Aghion and Blanchard (1994), by $rV_U = \frac{\dot{N}}{U}(V_N - V_U) + \frac{dV_U}{dt}$, with V_N being the value of being employed in the private sector, itself given by $rV_N = w + \frac{dV_N}{dt}$. The efficiency-wage argument enters the considerations by postulating that private-sector firms will set a wage such that the $V_N = V_U + c$, for some $c \geq 0$. This in turn implies $\frac{dV_N}{dt} = \frac{dV_U}{dt}$ and the two equations given in this footnote can be easily rearranged – by simply taking the difference – to lead to the equation for private-sector wages given in the remark.

and Blanchard (1994). The relationship (1) now follows from inserting $w = b + c(r + \frac{\dot{N}}{U})$ and $Ub = (1 - U)z$ into $\dot{N} = a(y - z - w)$.

Aghion and Blanchard (1994, Section 3.1) assume that at the outset of transition, employment in the state sector drops from 1 to some $E(0) = E_0 < 1$, which implies an initial unemployment rate equal to $U(0) = 1 - E_0$. This value of $U(0)$ will, in general, not correspond to the optimal choice of the unemployment path that maximizes the net present value of output. Consequently, the optimal unemployment path will have a discontinuity at $t = 0$ and jump to the optimal value from $U(0)$ immediately. Only, when the initial unemployment rate corresponds to the optimal choice will the optimal path of the unemployment rate be as illustrated below in Figure 1. Since our focus here is on the duration and end of a transition process, we abstract from the possibility of a discontinuity at $t = 0$ by assuming that $U(0)$ is optimally chosen as well, or equivalently that state sector employment drops to $E(0) = 1 - U(0)^*$, with $U(0)^*$ denoting the optimal choice of initial unemployment.

The government is only concerned with efficiency and chooses employment in the state sector to maximize the present discounted value of output. The government's optimization problem is thus given by:

$$\max_{E(t)} \int_0^{\infty} \left[E(t)x + N(t)y - \frac{1}{2ar} (f(U(t)))^2 \right] e^{-rt} dt, \quad (2)$$

subject to:

$$\dot{N}(t) = f(U(t)), \quad (3)$$

$$N(0) = 0, \quad (4)$$

$$E(t) + N(t) + U(t) = 1 \quad (5)$$

and non-negativity of $E(t)$, $N(t)$ and $U(t)$.

Based on the identity $E(t) + N(t) + U(t) = 1$, one immediately observes that the problem can equivalently be formulated using $U(t)$ as control variable and eliminating $E(t)$, which leaves only $U(t)$ and $N(t)$ in both the objective function and the constraints.¹⁰ This equivalent formulation of the problem is given by:

$$\max_{U(t)} \int_0^{\infty} \left[(1 - N(t) - U(t))x + N(t)y - \frac{1}{2ar} (f(U(t)))^2 \right] e^{-rt} dt, \quad (6)$$

¹⁰We perform this substitution to have $U(t)$, postulated to be constant along optimal paths by Aghion and Blanchard (1994), as the control variable. The benefit of this reformulation is that it allows us to highlight the differences between the approximate and the exact solutions.

subject to:

$$\dot{N}(t) = f(U(t)), \quad (7)$$

$$N(0) = 0, \quad (8)$$

$$N(t) \in [0, 1], \quad (9)$$

$$N(t) + U(t) \leq 1 \quad (10)$$

and non-negativity of $U(t)$.

Note first that an optimal, in fact any, path must necessarily fulfill exactly one of the following two properties: There exists a $\tau < \infty$ such that $\tau = \inf_{t \geq 0} (N(t) + U(t) = 1)$ or condition (10) is not binding for any finite t . These two cases will be discussed separately below. Before doing so, an important property of the model is derived in Proposition 1.

Proposition 1 *Along any path, it holds that $N(t) < 1$ for all $t < \infty$.*

Proof: For values of $N(t)$ sufficiently close to 1, the largest possible value of $\dot{N}(t)$ is given by setting $U(t) = 1 - N(t)$. The ordinary differential equation $\dot{N}(t) = f(1 - N(t))$ has a stable steady state at $N = 1$, since $f(0) = 0$ and $\frac{df(1-N)}{dN} = -f'(1-N) < 0$ for $N = 1$. Given that $N(0) = 0$, it follows that $N(t) < 1$ for $t < \infty$. \square

Let us now investigate potential optimal paths, starting with the case that the constraint (10) becomes binding for the first time at some $\tau < \infty$. Given that state sector employment is monotonically non-increasing, it follows that for $t \geq \tau$ the control problem has a trivial optimal solution. Denote with $N(t, N_\tau)$ the solution to the differential equation $\dot{N}(t) = f(1 - N(t))$, solved over (τ, ∞) , with initial condition $N(\tau) = N_\tau$. Note next that it trivially holds that $\frac{\partial N(\tau, N_\tau)}{\partial N_\tau} = 1$ and also note that up to now both τ and N_τ are unspecified. The objective function of the optimization problem from τ onwards is given by:

$$V(\tau, N_\tau) = \int_\tau^\infty \left[N(t, N_\tau) y - \frac{1}{2ar} (f(1 - N(t, N_\tau)))^2 \right] e^{-rt} dt. \quad (11)$$

Note the following relationships for the partial derivatives of the objective function (11):

$$\frac{\partial V(\tau, N_\tau)}{\partial \tau} = - \left[N(\tau, N_\tau) y - \frac{1}{2ar} (f(1 - N(\tau, N_\tau)))^2 \right] e^{-r\tau}, \quad (12)$$

$$\begin{aligned}\frac{\partial V(\tau, N_\tau)}{\partial N_\tau} &= \int_\tau^\infty \left[y + \frac{1}{ar} f(1 - N(t, N_\tau)) f'(1 - N(t, N_\tau)) \right] e^{-rt} dt \\ &= \frac{y}{r} e^{-r\tau} + \int_\tau^\infty \left[\frac{1}{ar} f(1 - N(t, N_\tau)) f'(1 - N(t, N_\tau)) \right] e^{-rt} dt.\end{aligned}\quad (13)$$

The optimization problem corresponding to the case considered can be rewritten as a *scrap value problem with free terminal time*, i. e., as a problem where τ is to be chosen optimally as well:

$$\max_{U(t) \in [0,1], \tau \in [0, \infty)} \left[\int_0^\tau \left[(1 - N(t) - U(t))x + N(t)y - \frac{1}{2ar} (f(U(t)))^2 \right] e^{-rt} dt + V(\tau, N_\tau) \right], \quad (14)$$

subject to (7), (9) and (10).

Problems of this type are studied in Seierstad and Sydsæter (1987, Theorem 3 and Note 2, p. 182–184), which provide necessary conditions for optimality.¹¹ The (current-value) Hamiltonian corresponding to this problem is given by $H(N, U, \mu, \tau) = (1 - N - U)x + Ny - \frac{1}{2ar}(f(U))^2 + \mu f(U)$, where we ignore, for brevity, the other constraints, (9) and (10), and the associated multipliers. It is straightforward but cumbersome to present the solution including these additional terms in the Lagrangean.¹²

Necessary conditions for optimality are given by:

$$-x - \frac{1}{ar} f(U) f'(U) + \mu f'(U) = 0, \quad (15)$$

$$\dot{\mu} = r\mu + x - y. \quad (16)$$

Furthermore, the following transversality condition has to hold:

$$\mu(\tau) e^{-r\tau} = \frac{\partial V(\tau, N_\tau)}{\partial N_\tau}. \quad (17)$$

The optimal terminal time τ is found from:

$$H(N, U, \mu, \tau) e^{-r\tau} + \frac{\partial V(\tau, N_\tau)}{\partial \tau} = 0. \quad (18)$$

¹¹To be precise we arrive at this type of problem only after verifying that the additional constraints – see the following Footnote 12 – are not binding. Problems with these additional constraints considered, i. e., with mixed and pure state constraints are discussed in Seierstad and Sydsæter (1987, Chapter 6).

¹²We use the terminology of Seierstad and Sydsæter (1987) and refer to the Hamiltonian augmented by the additional constraints as Lagrangean. It can be shown that these constraints are not binding, except possibly at $t = 0$ and $t = \infty$. More specifically, it can be shown that the only possible case where any other constraint than $U(t) + N(t) \leq 1$ is binding for $t < \infty$ is the case where $U(0) = 1$, in which case $\tau = 0$.

Equation (16) gives the following solution for $\mu(t)$:

$$\mu(t) = \frac{y-x}{r} + Ke^{rt}, \quad (19)$$

with K a constant whose value has to be determined from the transversality condition (17).

Remark 2 *Aghion and Blanchard (1994) derive an approximately optimal solution with a constant unemployment rate by setting $K = 0$ in the above equation (19). This clearly implies a constant value $\mu = \mu(t) = \frac{y-x}{r}$ of the costate variable. Inserting a constant value of the costate variable $\mu(t) = \frac{y-x}{r}$ into (15) then leads to a constant unemployment rate path, i. e., to the solution proposed in Aghion and Blanchard (1994, (26), p. 309).*

As noted in Aghion and Blanchard (1994), constant unemployment rate paths cannot be the exact solution for all values of t , since due to private sector job creation (which happens at a constant rate with a constant unemployment rate) at some point the unemployment rate has to decline. We show below, however, that the optimal unemployment rate is not constant already before the end of transition. This stems from the fact, discussed next, that along optimal paths K is positive, which leads to a non-constant costate variable and to a non-constant optimal unemployment rate path.

Let us next determine whether K is equal to zero or not for optimal paths. This question can be answered by inserting (13) and (19) into the transversality condition (17). After some rearrangements this yields:

$$Ke^{r\tau} = \frac{x}{r} + \frac{1}{ar} \int_{\tau}^{\infty} [f(1 - N(t, N_{\tau})) f'(1 - N(t, N_{\tau}))] e^{-r(t-\tau)} dt. \quad (20)$$

Signing K , thus, requires to sign the term in the square brackets in (20), for which the following proposition is helpful.

Proposition 2 *Along an optimal path, $f'(U(t)) > 0$ for all $t > 0$.*

Proof: First, note that for any choice of τ and N_{τ} there is an interval $[\tau + d, \infty)$ such that $f'(1 - N(t, N_{\tau})) = f'(U(t)) > 0$ for all $t \in [\tau + d, \infty)$. This is a straightforward implication of $U(t)$ tending to zero as $N(t)$ goes to 1. In particular, this implies that paths where $f'(U(t)) > 0$ for all t are always feasible if $U(t)$ is chosen to be small enough. Second, note that for every \widehat{U} such that $f'(\widehat{U}) < 0$, there is a value $\widetilde{U} < \widehat{U}$ such that $f(\widetilde{U}) = f(\widehat{U})$ and

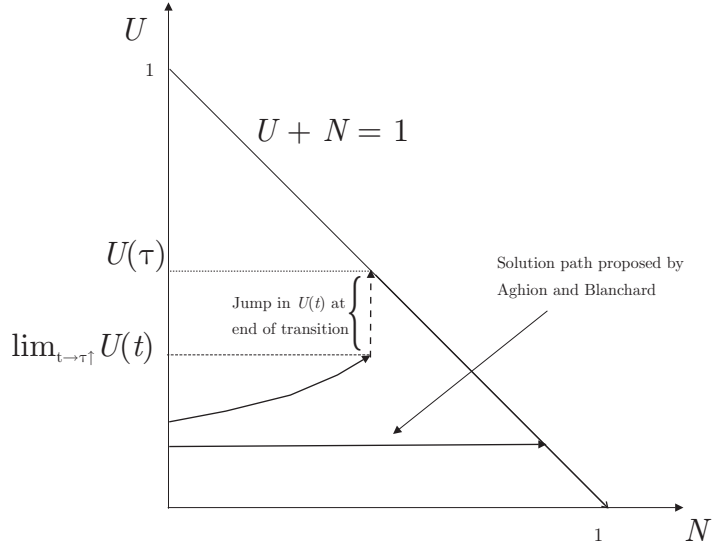


Figure 1: The transition process in (N, U) -space. Unemployment increases incrementally as a function of time until time τ , when the state sector is closed, the remaining workers are laid off and the unemployment rate jumps to a higher level $U(\tau)$.

$f'(\tilde{U}) > 0$.¹³ Since \tilde{U} and \hat{U} imply the same rate of job creation, but higher values of U are more costly, it follows that for the optimal choice of U it will always hold that $f'(U(t)) > 0$. Taken together these two facts imply that it is always possible to choose paths such that $f'(U(t)) > 0$ for all $t > 0$ and it is not optimal to choose other paths. \square

Proposition 2 implies that the second term in the square bracket in the integral in (20), $f'(1 - N(t, N_\tau))$, is positive and hence – since along optimal paths $\dot{N}(t) = f(1 - N(t, N_\tau)) > 0$ – the right-hand side in (20) is positive. Consequently, it follows that K is positive. This implies that $\mu(t)$ is not constant over time and consequently the optimal unemployment rate is also not constant over time. In fact, it follows that the optimal unemployment rate increases over time until τ , which is to be determined from equation (18). The fact that $K > 0$ implies that $\mu(t) > \frac{y-x}{r}$ for all $t < \tau$, which in turn implies that the optimal value of the unemployment rate $U(t)$ is – for all time points – higher than the approximately optimal unemployment rate derived in Aghion and Blanchard (1994, Section 6.4). This implies a fortiori that the optimal duration of transition is shorter than suggested by Aghion and Blanchard.

¹³This, of course, only applies to values $\hat{U} \leq \frac{y-b-rc}{y-rc}$, compare Footnote 8.

Note that there is another interesting feature of the optimal solution: The optimal unemployment rate is discontinuous at time τ , with the optimal path for the unemployment rate as illustrated in Figure 1 and the result derived analytically below.

Proposition 3 *For an optimal path of the unemployment rate, it holds that $\lim_{t \rightarrow \tau^-} U(t) < 1 - N(t)$. This implies that optimal unemployment $U(t)$ is discontinuous at τ .*

Proof: The proof is by contradiction, therefore assume that $\lim_{t \rightarrow \tau^-} U(t) = 1 - N(t)$. Then equations (12) and (18) imply that $\mu(\tau)f(1 - N(\tau)) = 0$. This in turn implies, since $N(\tau) < 1$ (which follows from Proposition 1), that $\mu(\tau) = 0$. Then equation (19) implies that $K < 0$, since $y > x$ by assumption. However, $K < 0$ is in contradiction with (20). \square

To complete the analysis, it remains to show that the second case, with condition (10) not binding for any finite t , cannot lead to optimal paths. Note first that, also in this case, $K \neq 0$, because $K = 0$ implies a constant unemployment rate (compare Remark 2). This follows from inserting (19) in (15), both of which now have to hold for all $t \geq 0$ for optimal paths. Since a constant unemployment rate implies a constant job creation rate, eventually the unemployment rate has to decrease because of a constant population size normalized to one. Thus, $K \neq 0$. This implies that $\mu(t)$ diverges to either plus or minus infinity, depending upon the sign of K . However, such a path of $\mu(t)$ cannot fulfill the necessary condition (15) for all $t \geq 0$, since both $f(U)$ and $f'(U)$ are bounded for $U \in [0, 1)$. This shows that paths with condition (10) not binding for any finite t cannot be optimal.

3 Conclusions

The discussed result has, which will not surprise experts in the field, close relations to problems of exhaustible-resource extraction. In the transition paper of Aghion and Blanchard (1994), the “mined resource stock” are the workers in the state sector, with mining here amounting to laying off. It is a typical feature of such models that the combination of discounting and the fact that a finite non-renewable resource stock is (eventually) depleted gives rise to optimal *extraction* paths that are non-constant over time.¹⁴

¹⁴An obvious – merely formal – difference between the transition process and a resource extraction problem is that the process of mining a resource yields instantaneous benefits, whereas in the transition model, mining (i.e., sending people into unemployment) is instantaneously costly, i.e., has negative benefits. This explains why models of exhaustible-resource extraction imply that resources should be optimally mined at a decreasing

As mentioned in the previous section, the formal term for the type of problem considered is scrap value problem with free terminal time. The discussion around the green transition often, at least in the general public and political discussion, implicitly or explicitly discusses problems with *exogenously chosen terminal time*, e. g., net zero emissions by 2050. This type of terminal time choice, of course, induces – ceteris paribus – resource owners (e. g., fossil fuel-producing countries) to exhaust their stock of resources until this point in time, which can be expected to lead to more “aggressive” extraction paths than without such a terminal time. This is clear, since a de-facto ban of fossil fuel usage from a certain point in time onwards amounts to setting the scrap value to zero from this point in time onwards. Correspondingly, fossil fuel-producing countries will *frontload* resource extraction (this supply response is, e. g., a key argument in Sinn, 2012). This, of course, casts doubt on the optimality – and in particular the *incentive compatibility* – of simple (time step-specified) decarbonization paths as discussed in public policy that, more often than not, ignore supply-side responses. This discussion exemplifies again that optimal transition processes need to be defined in terms of both optimal paths and optimal duration, in addition to – depending upon problem – zooming in on key aspects relevant for the problem at hand.

A more general observation we want to make with this note is – in addition to the obvious point of presenting the exact solution of Aghion and Blanchard (1994, Section 6.4) – that the analysis of economic transition processes may benefit by borrowing insights from resource economics. For transition processes involving resource (non-)extraction, like, e. g., the green transition, this appears obvious, but analogies prevail also in a broader sense.

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rate, whereas the transition model prescribes that unemployment should increase over the transition period $[0, \tau)$. This feature emerges in models that focus on a single mechanism and may be overturned in more complex models that allow for *substitution* between different factors or technologies. Hassler *et al.* (2022) present a model in which the optimal usage of a scarce resource increases over time, with low usage at the beginning being driven by a “lack” of human and physical capital. Two key classical – very insightful – contributions to resource extraction problems, Hotelling (1931) and Dasgupta and Heal (1974), have already clarified many issues.

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